

Stability of Collusion and Vertical Differentiation

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Abstract

We study the stability of collusion in a vertically differentiated duopoly industry. We consider a general framework that does not require restrictive assumptions that are common in the literature, such as full market coverage or fixed market shares. We study cases in which collusion can be sustained through side payments, and in which such payments are not feasible. In the latter case, we find that collusion becomes easier to sustain as the degree of differentiation increases. Moreover, the low-quality firm is more likely to deviate from the collusive agreement, hence determines cartel stability. With side payments, the same is true only for low degrees of differentiation, and incentives of the high-quality firm becomes more important if the degree of differentiation is sufficiently high. We also find that collusion under side payments is more stable for low levels of quality differentiation than when such payments are not allowed, while the opposite holds for high degrees of vertical differentiation. Therefore, side payments can become a factor of cartel destabilization for high degrees of quality differentiation.

Keywords: Collusion; Vertical differentiation; Side payments. JEL Classifications: D43; L13; L41.

1 Introduction

The theory of collusive agreements have been traditionally developed in symmetric environments such that market participants are identical in all aspects. While the issue of asymmetries among firms is critical in practice, collusion in such environments have been the subject of a relatively small literature. The traditional view states that asymmetries in costs or product characteristics hinder coordination on collusive pricing. A small literature has studied collusion under cost asymmetries (Rothschild, 1999; Miklòs-Thal, 2011) and under horizontal product differentiation (Chang, 1991; Vasconcelos, 2005). Theoretical

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work has also verified the traditional view in vertically differentiated industries (Häckner, 1994; Symeonidis, 1999).

The literature on collusion in vertically differentiated markets makes simplifying assumptions that are highly restrictive. It is common to assume that the market is fully covered, and that the market shares of market participants are the same under collusion and competition (Bos and Marini, 2019). Both of these assumptions neglect important incentives that may be important for collusion and deviation strategies of market participants. The assumption of full market coverage assumes away cases in which a firm can sell exclusively to the highest-valuation consumers. This also leads to a case in which some competition with the lowest-quality firms must be retained. Similarly, the assumption of fixed market shares prevent profitable deviation strategies. These assumptions are restrictive since having access to higher-valuation customers is disproportionately beneficial under vertical differentiation, and deviations may be targeted to increase market shares. As a result, our understanding of collusive practices in vertically differentiated industries remains insufficient.

In this article, we investigate the relationship between collusion and vertical differentiation in a general environment that does not require these assumptions. We do this exercise with and without the possibility side-payments, where the previous literature has focused on the latter. Accordingly, we add a new dimension related to the relationship between collusion and product differentiation: we argue that it is both the degree of vertical differentiation and the collusive technology in place -namely, whether or not side payments are allowed- that affect the sustainability of collusion. Without inter-firm payments, we find the novel result of a positive relationship between product differentiation and the stability of collusion; that is, collusion becomes more sustainable the degree of vertical differentiation increases. Moreover, the collusive agreement becomes more stable in the asymmetric case when there is a leader and a follower in the market rather than when both firms sell identical products for higher degrees of vertical differentiation. This result contrasts with the case where side payments are allowed when the collusive outcome is always more stable in the asymmetric case.

Moreover, we complement the literature by showcasing that it is either the high (Häckner, 1994) or the low-quality firm (Symeonidis, 1999; Bos and Marini, 2019) that has stronger incentives to deviate from the collusive agreement. In line with Bos and Marini (2019) and Symeonidis (1999), we show that it is the low-quality firm that has stronger incentives to cheat when side payments are not feasible, or banned. In contrast, when inter-firm payments are allowed, the low-quality firm is more likely to deviate from the collusive agreement for lower degrees of product differentiation, and the high-quality firm deviates for larger degrees of differentiation. We also provide a comparison of the stability of collusion with and without side payments. We find that for low levels of quality differentiation, side payments make collusion more stable, but they can work as a factor of destabilisation for higher levels of quality differences between the competitors.

2 The Model

There are two firms, A and B, who interact repeatedly in the same market over an infinite, discrete time horizon. The two firms supply a single product in each period t , whose quality is given by q_i , $i = A, B$, and is common knowledge. Marginal costs of production are zero for both firms. They have a common discount factor $\delta \in (0, 1)$.

Consumers are heterogenous in their valuations θ of product quality q_i . The mass of consumers is normalised to 1, and θ is assumed to be distributed uniformly on $[0, 1]$. Consumers have unit demands for the product, with utility functions given by

$$U(\theta) = \begin{cases} \theta q_i - p_i & \text{when buying from firm } i \\ 0 & \text{when not buying,} \end{cases} \quad (1)$$

where p_i is the price set by firm i .

There are two possibilities: Firms may offer products of the same quality at all periods (symmetric case). In such a case, product quality is denoted by q_0 . Alternatively, one firm (hereafter the leader) produces a better quality product q_L than its competitor (hereafter the follower), whose quality is denoted by q_F . Let q_L and q_F be constant over time and that $q_L > q_F$ (asymmetric case).

We consider the case where the two firms are colluding in prices at the beginning of the game following grim trigger strategies: they continue to cooperate until a single deviation occurs. Deviation by one firm is observed by its rival within the period in which it occurs. If a deviation occurs, firms revert back to the Nash equilibrium and compete in prices. The sustainability of the collusive outcome depends on the unilateral firms' incentives; that is, for each firm i , it should be that:

$$\Pi_i^c \geq (1 - \delta)\Pi_i^d - \delta\Pi_i^*, \quad (2)$$

where Π_i^c is the per period profit under collusion, Π_i^d is the one-period deviation profit and Π_i^* is the per period profit under price competition, for each firm i . The condition (2) implies that firm i does not have incentives to deviate for a discount factor equal or above a critical value $\hat{\delta}_i$ such that

$$\delta \geq \hat{\delta}_i = \frac{\Pi_i^d - \Pi_i^c}{\Pi_i^d - \Pi_i^*}. \quad (3)$$

3 Analysis

We consider the case where firms compete and not collude as the benchmark. When firms' products have the same quality q_0 , standard Bertrand competition leads to zero prices and profits, while the market is shared equally by the two firms:

$$\begin{aligned} p_A = p_B = p_0^* = 0 \\ \text{and } \Pi_A = \Pi_B = \Pi_0^* = 0 \end{aligned} \quad (4)$$

In the asymmetric case, the indifference consumer $\hat{\theta}$ is the one such that

$$\hat{\theta} = \frac{p_L - p_F}{q_L - q_F}, \quad (5)$$

where p_L refers to the price set by the leader and the p_F refers to the price of the follower.

The follower serves demand $\max\{\min\{\hat{\theta}, 1\} - \underline{\theta}, 0\}$, where, $\underline{\theta}$ is defined by

$$\underline{\theta}q_F - p_F = 0 \Rightarrow \underline{\theta} = \frac{p_F}{q_F}. \quad (6)$$

The demand for the leader is given by $\max\{0, 1 - \max\{\bar{\theta}, \hat{\theta}\}\}$, where,

$$\bar{\theta}q_L - p_L = 0 \Rightarrow \bar{\theta} = \frac{p_L}{q_L}. \quad (7)$$

Note that in each equilibrium (p_L^e, p_F^e) we can discriminate between two cases:

- if $\frac{p_L^e}{p_F^e} > \frac{q_L}{q_F}$, then $\hat{\theta}^e > \bar{\theta}^e > \underline{\theta}^e > 0$, so, the leader serves demand $\max\{1 - \hat{\theta}^e, 0\}$ and the follower serves demand $\max\{\min\{\hat{\theta}^e, 1\} - \underline{\theta}^e, 0\}$
- otherwise, $\underline{\theta}^e \geq \bar{\theta}^e \geq \hat{\theta}^e$ and the leader serves demand $\max\{1 - \bar{\theta}^e, 0\}$, while, the follower does not face any positive demand.

We assume that in equilibrium p_L^e, p_F^e it is $1 > \hat{\theta}^e \geq \bar{\theta}^e \geq \underline{\theta}^e > 0$, so, the two firms solve simultaneously the following double maximization problem:

$$\begin{aligned} \Pi_F^* &= \max_{p_F} \left\{ p_F \left(\hat{\theta} - \underline{\theta} \right) \right\} \\ \Pi_L^* &= \max_{p_L} \left\{ p_L \left(1 - \hat{\theta} \right) \right\}, \end{aligned}$$

and then we check if the derived equilibrium prices are indeed interior to our assumption conditions. The competitive equilibrium in this asymmetric case will be:

$$\begin{aligned} \hat{\theta}^* &= \frac{2q_L - q_F}{4q_L - q_F} \\ p_F^* &= \frac{q_F(q_L - q_F)}{4q_L - q_F} \quad \& \quad \Pi_F^* = \frac{q_L q_F (q_L - q_F)}{(4q_L - q_F)^2} \\ p_L^* &= \frac{2q_L(q_L - q_F)}{4q_L - q_F} \quad \& \quad \Pi_L^* = \frac{4q_L^2(q_L - q_F)}{(4q_L - q_F)^2}. \end{aligned} \quad (8)$$

Note that $1 > \hat{\theta}^* > \bar{\theta}^* > \underline{\theta}^* > 0$ holds in equilibrium and that all prices and profits are strictly increasing in q_L . Moreover, the follower has positive equilibrium profits despite its product quality disadvantage. The reason for this is that the quality differentiation creates two variants of the product that in equilibrium are priced differently and in this way it allows the follower to raise its price above its marginal cost. However, it is the leader that benefits the most from an increase in q_L , since $\frac{d\Pi_L}{dq_L} > 2\frac{d\Pi_F}{dq_L} > 0$.

When firms collude, their profitability depends on the quality difference of the two firms. We model optimal collusion as a state at which

- firms maximize their joint profit given their production qualities,
- they share between each other the collusive joint profit in a Pareto optimal way through a Nash bargaining process.

We first compute the critical discount factor for the sustainability of collusion in the symmetric case, $\hat{\delta}_0$. We then study the asymmetric case under two different profit sharing mechanisms: when side payments are allowed and when they are not. Optimal deviation profits are computed in a standard and intuitive way, in which the deviating firm maximises its one period deviation profit given the collusive equilibrium price of its rival.

3.1 Collusion under symmetry

In the symmetric case that $q_A = q_B = q_0$, the demand is $1 - \frac{p}{q_0}$ and the profit maximization can be expressed as:

$$\Pi_i = \max_{p_0} \left\{ p_0 \left(1 - \frac{p_0}{q_0} \right) \right\}.$$

Solving this maximization problem and considering that collusive profits are equally shared by the two firms through Nash bargaining, we conclude that

$$\begin{aligned} p_A = p_B = p_0^c &= \frac{q_0}{2} \\ 2\Pi_0^c &= \frac{q_0}{4}, \text{ or } \Pi_A = \Pi_B = \Pi_0^c = \frac{q_0}{8}. \end{aligned} \quad (9)$$

Studying now the sustainability of collusion, we observe that the optimal deviation from the collusive agreement by each firm with quality q_0 is to set price marginally below p_0^c and realize profit $\Pi_0^d = \frac{q_0}{4}$, so, following (3) and (4):

$$\hat{\delta}_0 = \frac{1}{2}.$$

3.2 Collusion under asymmetry with side payments

In the asymmetric case, when we allow for side payments between the two firms, we follow a similar approach as described in the derivation of competitive equilibrium. Hence, the maximization problem can be defined as:

$$\Pi_{\text{joint}}^c = \max_{p_F, p_L} \left\{ p_F (\hat{\theta} - \underline{\theta}) + p_L (1 - \hat{\theta}) \right\}. \quad (10)$$

Maximization of the joint profit leads to the following equilibrium:

$$\begin{aligned} p_F^c &= \frac{q_F}{2} \\ p_L^c &= \frac{q_L}{2} \\ \Pi_{\text{joint}}^c &= \frac{q_L}{4} \end{aligned} \quad (11)$$

Equilibrium prices in this case imply that $\hat{\theta}^c = \frac{1}{2}$. Moreover, it is easy to see that $\hat{\theta}^c = \underline{\theta}^c = \bar{\theta}^c$, so, only the leader produces and serves demand $1 - \hat{\theta}^c$.

Collusion is only sustained if firms can divide the collusive total profit in such a way that they are both better off under collusion.¹ As a profit sharing mechanism through side payments from the leader to the follower, we consider the following Nash bargaining game where the two firms have equal bargaining power and which takes the following form:

$$\begin{aligned} \max_{\Pi_L^c, \Pi_F^c} \{(\Pi_L^c - \Pi_L)(\Pi_F^c - \Pi_F)\} \\ \text{s.t. } \Pi_L^c + \Pi_F^c = \Pi_{\text{joint}}^c. \end{aligned}$$

It leads to equilibrium collusive profits

$$\begin{aligned} \Pi_F^c &= \frac{3q_L q_0}{8(4q_L - q_0)} \\ \Pi_L^c &= \frac{8q_L^2 - 5q_L q_0}{8(4q_L - q_0)}. \end{aligned} \quad (12)$$

Note that both Π_F^c is strictly decreasing in q_L , while Π_L^c are strictly increasing in q_L . So, $\Pi_L^c - \Pi_F^c$ is strictly increasing in q_L . Moreover, $\Pi_F^c > \Pi_F^*$ and $\Pi_L^c > \Pi_L^*$, $\forall q_L, q_0$, with $q_L > q_0 > 0$. So, the collusive participation constraints are satisfied by the outcome of the Nash bargaining game.

It is easy to see that the leader's optimal deviation from the collusive agreement is to refuse making any side payment to the follower and to get for one period profit, so: $p_L^d = p_L^c$ and $\Pi_L^d = \Pi_{\text{joint}}^c$. For all the remaining periods, the two firms earn the Bertrand/Nash profits given by (12).

The follower's optimal deviation will be to select the price that maximizes its profit given leaders pricing p_L^c . This leads to:

$$p_F^d = \begin{cases} q_F - \frac{q_L}{2} & \text{if } 1 < \frac{q_L}{q_F} < \frac{3}{2}, \\ \frac{q_F}{4} & \text{if } \frac{3}{2} \leq \frac{q_L}{q_F}. \end{cases}$$

and

$$\Pi_F^d = \begin{cases} \frac{q_L(2q_F - q_L)}{4q_F} & \text{if } 1 < \frac{q_L}{q_F} < \frac{3}{2}, \\ \frac{q_L q_F}{16(q_L - q_F)} & \text{if } \frac{3}{2} \leq \frac{q_L}{q_F}. \end{cases} \quad (13)$$

Note that Π_F^d (Π_L^d) is strictly decreasing (increasing) in q_L and that $\Pi_L^d > \Pi_L^c$, $\forall q_L > q_F > 0$. However, in the case of the follower, $\Pi_F^d \geq \Pi_F^c$, only if $\frac{q_L}{q_F} \geq \frac{5}{2}$. So, for high quality differentiation such that $\frac{q_L}{q_F} > \frac{5}{2}$, the follower does not have any incentive to deviate from the collusive agreement

The critical discount factors (3) for the leader and the follower are:

¹We assume that if a firm gets the same profit under competition and under collusion, it always prefers to compete.

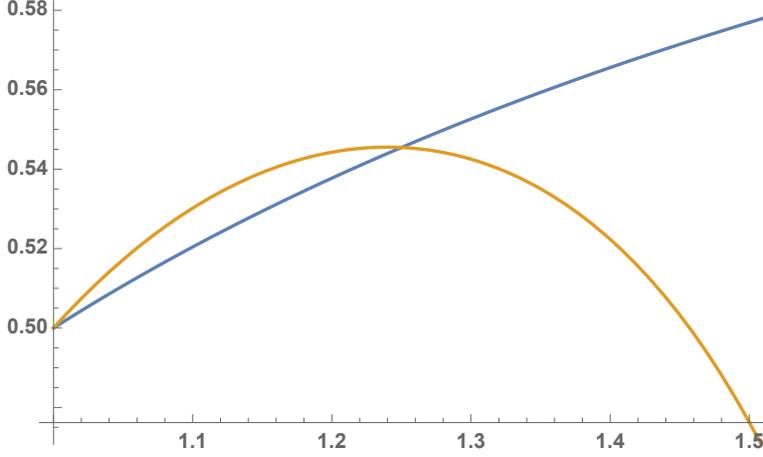


Figure 1: The critical discount factors of the leader and the follower as a function of quality differentiation k under side payments.

$$\hat{\delta}_L^{sp} = \frac{3}{4} \left(1 - \frac{3}{1 + 8k} \right), \quad (14)$$

and

$$\hat{\delta}_F^{sp} = \begin{cases} 1 - \frac{4k+5}{12-42k+80k^2-32k^3} & \text{if } 1 < k < \frac{3}{2}, \\ \frac{(4k-1)(5-2k)}{3(8k-5)} & \text{if } \frac{3}{2} \leq k, \end{cases} \quad (15)$$

where, $k = \frac{q_L}{q_F}$. The critical discount factor $\hat{\delta}_L^{sp}$ is strictly increasing and concave in k , namely, the leader finds increasingly attractive to deviate as the quality differentiation between the two firms increases, but these increasing deviation incentives exhibit diminishing returns. The critical discount factor of the follower, $\hat{\delta}_F^{sp}$ exhibits an inverted-U relationship with quality differentiation k . For low levels of k such that $k < 1.234$ the follower have increasing incentives to deviate as k increases. There is a value $k^{cr} = 1.234$ at which the follower's deviation incentives are maximised, while, following that point, further increases in k decrease its incentives for deviation. Figure 1 depicts the critical discount factors for the leader and the follower as a function of k .

The following two propositions summarise the main two results of this section:

Proposition 1. *In collusion under side payments,*

- *for low quality differentiation, $1 < k < \frac{5}{4}$, it is the follower that has more incentives to deviate from the collusive agreement, $\hat{\delta}_F^{sp} > \hat{\delta}_L^{sp}$,*
- *for high quality differentiation such that $\frac{5}{4} < k$, it is the leader that has more incentives to deviate from the collusive agreement, $\hat{\delta}_F^{sp} < \hat{\delta}_L^{sp}$,*
- *When $k = \frac{5}{4}$, the leader and the follower have the same incentives to deviate, $\hat{\delta}_F^{sp} = \hat{\delta}_L^{sp}$.*

By comparing the sustainability of collusion in the symmetric and the asymmetric cases, we conclude that:

Proposition 2. *Collusion under side payment is more stable under symmetry, when $k = 1$, than under asymmetry, when $k > 1$.*

The stability of the collusive agreement in the asymmetric case is defined by $\delta_{cr}^{sp} = \max\{\hat{\delta}_F^{sp}, \hat{\delta}_L^{sp}\}$. It is easy to see that $\delta_{cr}^{sp} > \hat{\delta}_0$, for every $k \geq 1$.

3.3 Collusion under asymmetry without side payments

Antitrust law forbids direct (overt) monetary transfers in most jurisdictions. To be in line with this we consider now the case that side payments are not possible. Collusion then can be sustained only when both firms produce and have positive market shares. While the objective is again to maximise joint profit, the follower should have positive market share and adopt a pricing strategy such that it will be better off than its profit at the competitive state. So, joint profit maximisation is now constrained by the fact that p_L and p_F should be selected such that the follower will earn a profit at least as high as $\bar{\Pi}_F > \Pi_F^*$ in the absence of any direct payments. The optimal value of threshold $\bar{\Pi}_F$ is determined through Nash bargaining of the two firms. Profit maximisation (10) is now constrained by

$$p_F (\hat{\theta} - \underline{\theta}) \geq \bar{\Pi}_F$$

As in equilibrium this constraint will be binding, for computing the collusive equilibrium in this case we have to solve the system of equations: the first order condition for the leader at problem (10) and the binding constraint for the follower. This system has a solution only if $\bar{\Pi}_F < \frac{q_F}{16}$, which is²:

$$\begin{aligned} p_F^c(\bar{\Pi}_F) &= \frac{1}{4} \left(q_F + \sqrt{q_F} \sqrt{q_F - 16\bar{\Pi}_F} \right) \\ p_L^c(\bar{\Pi}_F) &= \frac{1}{2} q_L - \frac{1}{4} \left(q_F - \sqrt{q_F} \sqrt{q_F - 16\bar{\Pi}_F} \right) \end{aligned}$$

so, at the optimum, collusive profits for the leader and the follower are functions of $\bar{\Pi}_F$, $\Pi_F^c(\bar{\Pi}_F) = \bar{\Pi}_F$ and $\Pi_L^c(\bar{\Pi}_F)$. The optimal value of $\bar{\Pi}_F$ is determined by the following bargaining problem:

$$\max_{\bar{\Pi}_F} \{ (\Pi_L^c(\bar{\Pi}_F) - \Pi_L)(\bar{\Pi}_F - \Pi_F) \},$$

²The follower's binding constraint has two solutions for p_F . However, it can be shown that it is the highest one that maximises joint profit, and given the objective of collusion, this is the solution we consider in the maximisation problem.

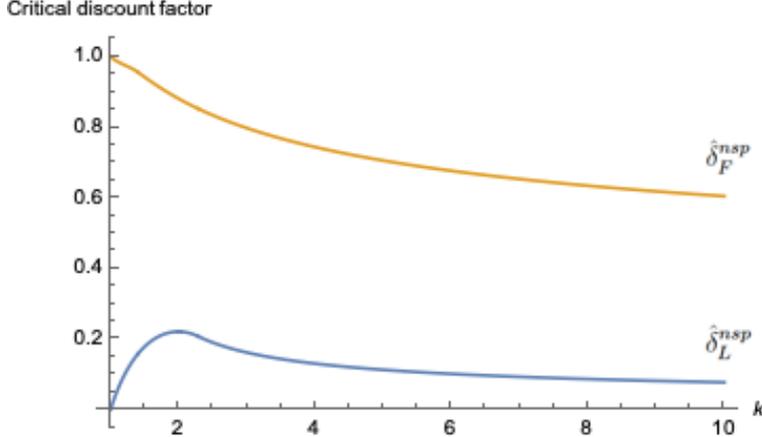


Figure 2: The critical discount factors of the leader and the follower as a function of quality differentiation k .

which leads to³:

$$\begin{aligned} \bar{\Pi}_F^{opt} = & -\frac{8}{288} \sqrt{\frac{q_F^5(10q_L - q_F)^2(q_F^3 - 5q_F^2q_L - 11q_Fq_L^2 + 96q_L^3)}{(4q_L - q_F)^8}} \\ & + \frac{8q_F(q_F^4 - 26q_F^3q_L + 202q_F^2q_L^2 - 672q_Fq_L^3 + 576q_L^4)}{288(4q_L - q_F)^4} \end{aligned}$$

The solution $\bar{\Pi}_F^{opt}$ allows to fully characterize collusive equilibrium prices $p_F^c(\bar{\Pi}_F^{opt})$, $p_L^c(\bar{\Pi}_F^{opt})$ and profits $\Pi_L^c(\bar{\Pi}_F^{opt})$, $\bar{\Pi}_F^{opt}$ for the leader and the follower, respectively. Collusive profit are strictly increasing and concave in k . Moreover, indeed, $\frac{q_F}{16} > \bar{\Pi}_F^{opt} > \Pi_F^*$, $\forall k > 1$.

We compute the optimal deviations for the leader and the follower following the same steps as in the side payments case. The only difference is that this time, since side payments are not possible, the leader deviates by changing its pricing strategy. Both firms select the price that maximise their profits taking the collusive pricing of their rival as given.

The derivation of the deviation profits allow us to compute the critical discount factors $\hat{\delta}_F^{nsp}$ and $\hat{\delta}_L^{nsp}$ from (3). They are depicted in Figure 2. The discount factor $\hat{\delta}_F^{nsp}$ is decreasing in quality differentiation $k = \frac{q_L}{q_F}$. In contrast, $\hat{\delta}_L^{nsp}$ is increasing in k for low levels of quality differentiation such that $k < 1.994$ and decreasing in k for $k > 1.994$.

Proposition 3. *If side payments are not allowed, the follower has higher incentives to deviate from the collusive agreement than the leader, $\hat{\delta}_F^{nsp} > \hat{\delta}_L^{nsp}$, $\forall k > 1$.*

The stability of collusion in this case is defined by the incentives of the follower to collude since, $\delta_{cr}^{nsp} = \max\{\hat{\delta}_F^{nsp}, \hat{\delta}_L^{nsp}\} = \hat{\delta}_F^{nsp}$. Comparing this critical value with $\hat{\delta}_0$ we conclude to the following proposition.

³The Nash bargaining problem admits two solutions for $\bar{\Pi}_F$. We select the one that is consistent with the joint profit maximisation problem.

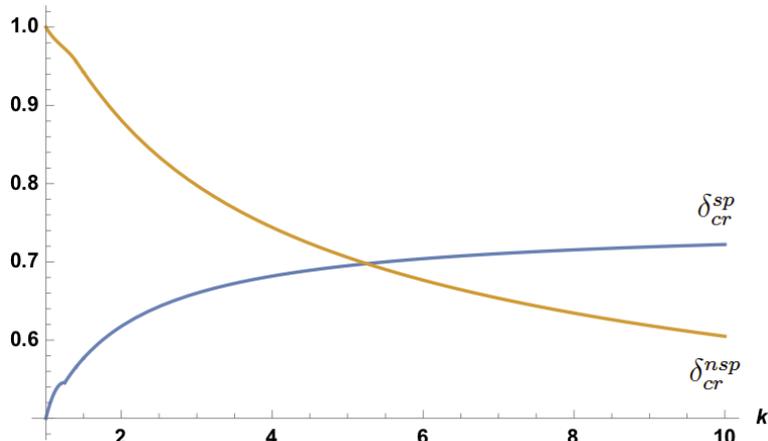


Figure 3: The critical discount factors for the stability of collusion with and without side payments as a function of quality differentiation k .

Proposition 4. *If side payments are not allowed, then*

- *for $1 < k < 27.734$, collusion is more stable under symmetry, when $k = 1$, than under asymmetry.*
- *for $k > 27.734$, collusion is more stable under asymmetry than under symmetry, when $k = 1$.*

So, the exact degree of quality differentiation determines whether quality asymmetry makes collusion more or less stable. For high levels of quality differentiation, asymmetry makes collusion more stable.

3.4 Prohibition of side payments and cartel stability

As antitrust rules prohibit side payments, by comparing the stability of collusion with and without side payments for different levels of quality differentiation k . In this way, we can approximate the impact of such an antitrust intervention on cartel stability.

Critical discount factors δ_{cr}^{sp} and δ_{cr}^{nsp} are depicted in Figure 3. The degree of quality differentiation is crucial of whether side payments lead to more or less stable collusion. Specifically,

Proposition 5. *For quality differentiations such that $k < 5.251$ collusion under side payments is more stable, $\delta_{cr}^{sp} > \delta_{cr}^{nsp}$. for quality differences such that $k > 5.251$ collusion without side payments is more stable, $\delta_{cr}^{sp} < \delta_{cr}^{nsp}$.*

So, for high levels of quality differentiation, the permission of side payments can lead to the destabilisation of the collusive agreement. The reason is that the leader has more incentives to deviate and capture the market by stopping providing the side payment to its conspirator. The high quality difference guarantees a high payment in the punishment-price competition-face that will follow. In constrast, without side payments, as both firms

produce and market their products in the collusive state, leader's deviation can never be as profitable as in the side payments case. That is particularly true for high quality differentiations.

3.5 Conclusion

This paper investigates the stability of collusion in a vertically differentiated duopoly. We relax restrictive assumptions that have been commonly used in the literature. We also differentiate cases in which side-payments among market participants are, and are not feasible. Our analysis reveals novel insights into collusion under vertical differentiation. We show that whether or not side-payments can be used as part of collusive strategies is important for market outcomes, and leads to different results. When side-payments are not feasible or are banned, we find that collusion becomes more sustainable as the market becomes further differentiated. This is in contrast to the previous literature that has found a negative relationship between the degree of differentiation and cartel stability.

Relaxing the assumptions of full market coverage and fixed market shares allow profitable deviation strategies for a low-quality firm, in which deviations give access to highest-valuation consumers who previously preferred the high-quality firm's product. As a result, a low-quality firm has stronger incentives to deviate than under full coverage and/or fixed market shares. Despite these additional incentives, the low-quality firm is not universally decisive for cartel stability, as the previous literature has argued. If side-payments can be used, this latter result holds for low degrees of differentiation, but incentives of the high-quality firm to deviate becomes stronger if the degree of differentiation is sufficiently high. This result clarifies an ambiguity regarding market players with greater incentives to destabilize cartels.

Our results are also informative regarding the role of side-payments in collusive arrangements. We find that side payments render collusion more stable compared to the case in which such payments are not allowed *only if* product qualities in the industry are sufficiently close to one another. If quality differences are high, the opposite holds, and banning side payments renders collusion more stable. This occurs because the firm with the critical discount factor changes across the two cases. For high quality differences, the high-quality firm becomes decisive for cartel stability in the side-payments case, and is more impatient compared to the (decisive) low-quality firm under a no side-payment regime.

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