RECAPTURE RATIOS IN MERGER ANALYSIS[1]

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Abstract. *Most quantitative tools for assessing potential competitive effects of mergers (e.g., upward pricing pressure indices and merger simulation models) rely heavily on recapture ratios. We show how recapture ratios can be approximated using simple microeconomic relationships and commonly-used parameters—namely, the products' sales, profit margins, and group elasticity of demand. We illustrate our approach for both horizontal and vertical product differentiation.*

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1. INTRODUCTION

There is a large literature developing quantitative tools for assessing potential competitive effects of mergers in differentiated products industries. These tools include *upward pricing pressure indices* (e.g., Werden, 1996; Farrell and Shapiro, 2010) and *merger simulation models* (e.g., Shapiro, 1996; Hausman et al., 2011).[2] These papers develop quantitative methods based on rigorous microeconomic theory and simple pre-merger parameters, such as profit margins and diversion ratios between competing products.

The initial objective of these methods was to provide competition authorities with systematic economic tools for screening mergers at the early stages of an investigation. However, despite their theoretical elegance and simplicity, these methods have often been difficult to apply in practice. The main reason is that diversion ratios, which are a core ingredient of these approaches, are usually unknown at the early stages of an investigation. Accordingly, these tools are applied only in a limited number of cases, and typically at a much later stage of the investigation, when reliable estimates of diversion ratios may become available.

To illustrate the difficulty in applying these tools at the early stages of a merger review, consider the *gross upward pricing pressure index* (GUPPI) commonly used in horizontal merger cases involving differentiated products. Calculating the GUPPI for one of the merging brands requires estimating the diversion ratio from that brand to the other merging brand. In principle, one could estimate the diversion ratio using data showing how sales have shifted between the two merging brands

[2] These tools have been extended in various ways (e.g., O'Brien and Salop, 2000; Goppelsroeder, Schinkel and Tuinstra, 2008; Salop and Moresi, 2009; Moresi, 2010; Jaffe and Weyl, 2013).

when the price of the first brand changed and all other prices remained constant. One also could estimate the diversion ratios between the merging brands using business documents of the merging firms, such as win/loss reports and survey data regarding customer switching patterns. However, it is often difficult to obtain and analyze such data at the early stages of a merger investigation.

In the absence of concrete estimates of diversion ratios, competition authorities in practice often assume that diversion ratios are proportional to market shares,[3] which corresponds to the Independence of Irrelevant Alternatives (IIA) assumption in logit models.[4] As an example, consider three competing products, A, B and C, and suppose that the market share of B is twice as large as the market share of C (say, 40% for B and 20% for C).[5] Under the assumption that diversion is proportional to market shares, if the price of product A were to increase unilaterally, then the number of customers that would leave A and switch to B is twice as large as the number of customers that would switch to C.

Yet, even where such *relative* diversion between "inside products" can be approximated in this way, the *absolute* level of the diversion ratios is usually difficult to determine, because it requires estimating the diversion ratio to "outside goods" or, equivalently, it requires estimating the *recapture ratio*.[6] The recapture ratio for a given product is the fraction of that product's lost sales (following a unilateral increase

[3] The proportionality assumption can be based on indicators of diversion that are different from market shares. For instance, in mergers involving mobile wireless carriers, proportionality is sometimes based on the carriers' shares of gross additions of subscribers or on the porting rates of phone numbers from individual carriers to their respective competitors. Our analysis is applicable in those cases as well.

[4] See, e.g., Hausman and McFadden (1984).

[5] We use the term "market share" loosely, since the products included in the analysis and used to calculate market shares need not necessarily constitute a "market" in a legal or antitrust economics sense.

[6] The *recapture ratio* is called "recapture percentage" in the U.S. Horizontal Merger Guidelines, and "retention ratio" or "group recapture ratio" or "aggregate diversion ratio" in the economics literature.

in the price of that product) that stays in the "market" rather than being diverted to outside goods. In short, estimating the individual diversion ratios—say, from A to B and from A to C—using the proportionality assumption requires knowledge of the recapture ratio (RR) for product A, that is, the total fraction of the sales lost by A that is recaptured by B and C.

The importance of estimating the RR can be illustrated using the previous numerical example. Suppose that diversion to outside goods is zero, and hence the RR for product A is 100%. Proportional diversion then implies that the diversion ratio from A to B is 66.7% and that from A to C is 33.3%. However, if instead the diversion ratio to outside goods were 30% (i.e., the RR were 70%), then proportionality would imply that the diversion ratio from A to B is only 46.7% (instead of 66.7%) and that from A to C is only 23.3% (instead of 33.3%). It follows that the common practice among antitrust economists of using *ad hoc* assumptions on RRs (e.g., assuming 100% or 70%) can lead to estimated post-merger price effects that significantly misrepresent the likely competitive effects of a merger. For instance, simply assuming zero diversion to outside goods (as is often done in practice for analytical convenience) will bias post-merger price increases upwards and can easily lead to predictions that overstate the actual price effects of a merger by 100% or more.[7]

This paper shows how RRs can be approximated directly using simple microeconomic relationships and commonly-used parameters—namely, the products' sales, profit margins, and group elasticity of demand. Given a broad set of firms and

[7] Suppose that two out of three symmetric firms merge in an industry with an actual RR of 66.7%. Thus, the actual diversion ratio between the two merging products equals 33.3%. If instead one were to assume a RR of 100% (i.e., zero diversion to outside goods), the diversion ratio between the two merging products would equal 50%. Using Werden (1996), one can check that overstating the RR in this way would lead to overstating the anticompetitive effects of the merger by 100%.

competing products, the sales and margins of each product often can be approximated reasonably well using company documents provided by the merging firms. If company documents and industry studies do not provide a good approximation of the group elasticity of demand, one can use either -1 as a reasonable prior or, as suggested by Clements (2008), -0.5 in industries involving commodities. Our method for estimating RRs thus facilitates the estimation of diversion ratios and the assessment of potential unilateral effects of mergers, using either upward pricing pressure indices or merger simulation models (e.g., Werden, 1996; Hausman et al., 2011).[8]

Our analysis also can be useful even where reliable customer switching data become available at the more advanced stages of an investigation, because switching data typically do not contain much information about substitution with outside goods. For example, while most mobile wireless carriers possess a wealth of information on diversion to and from other mobile wireless carriers, they typically have very little information about diversion to and from outside goods (or non-consumption).

For these reasons, it is important for merger analysis to be able to approximate RRs in a sensible and rigorous way. In order to do so, this paper explores the general relationship between RRs and the group elasticity of demand.[9] As it turns out, exploiting this economic relationship permits to approximate RRs in a sensible way and with simple, observable parameters that are typically available even at the early stages of an investigation.

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[8] Our analysis also is useful in other areas of antitrust economics where the application of quantitative techniques relies on the availability of diversion data. For example, diversion ratios play a key role in the analysis of vertical integration (e.g., Moresi and Salop, 2013).

[9] Formally, the group elasticity of demand measures the response of *total* demand for the group of products under consideration following a proportional increase in the prices of *all* products in the group.

Specifically, this paper derives analytical solutions both for the case of symmetric RRs and for the case of asymmetric RRs. For asymmetric RRs, we consider the natural benchmarks of proportional diversion to outside goods and vertical product differentiation). In these circumstances, RRs can be directly inferred using no more information than the pre-merger products' shares, profit margins, and group elasticity of demand. This substantially facilitates the application of quantitative tools to assess the economic effects of mergers when data availability is limited.

The paper is structured as follows. Section 2 introduces the model, which builds on the standard model of price competition with differentiated products, and determines the elementary relationship between RRs and group elasticity of demand. Section 3 uses this relationship to analyze symmetric RRs and derives a closed-form solution for this case. Sections 4 and 5 then consider the alternative scenario of asymmetric RRs. Specifically, Section 4 derives a solution for the case of proportional RRs, whereas Section 5 extends the analysis to the case of vertical product differentiation. Section 6 discusses the implications of our results for applied merger analysis, and Section 7 concludes.

2. THE MODEL

There are $N \geq 2$ differentiated products that are (imperfect) substitutes from the perspective of customers. The demand for product k is given by

$$q_k = D_k(p_1, \dots, p_N) \qquad \text{for } k = 1, \dots, N \tag{1}$$

where q_k denotes the quantity demanded, D_k denotes the demand function, and p_j denotes the price of product j. The own-price elasticity of product k is given by

$$\varepsilon_k = \frac{\partial D_k}{\partial p_k} \frac{p_k}{q_k}. \tag{2}$$

The total demand for the N products is given by

$$q = \sum_{k=1}^{N} D_k(p_1, \dots, p_N) \tag{3}$$

and the group elasticity of demand (with respect to a proportional increase in all prices) is given by[10]

$$E = \frac{t}{q}\frac{d}{dt}\sum_{k=1}^{N} D_k(tp_1, \dots, tp_N)\,|_{t=1} = \sum_{k=1}^{N}\sum_{j=1}^{N}\frac{\partial D_k}{\partial p_j}\frac{p_j}{q}. \tag{4}$$

The diversion ratio from product j to product k is given by

$$\delta_{jk} = -\frac{\partial D_k/\partial p_j}{\partial D_j/\partial p_j}. \tag{5}$$

The RR for product j is the total diversion ratio from j to all other products

$$R_j = \sum_{k \neq j} \delta_{jk}. \tag{6}$$

Note that the diversion ratio from product j to "outside goods" equals $1 - R_j$. Different products may have different RRs and thus different diversion ratios to outside goods.

Using the above definitions, we are now ready to consider the relationship between elasticities and RRs. Note first that equation (4) can be rewritten as

$$E = \sum_{j=1}^{N}\left(\frac{p_j}{q}\sum_{k=1}^{N}\frac{\partial D_k}{\partial p_j}\right) \tag{7}$$

by reversing the order of summation in (4) and then factorizing p_j/q. Using (5), equation (7) can be rewritten as

$$E = \sum_{j=1}^{N}\left(\frac{p_j}{q}\frac{\partial D_j}{\partial p_j}\left[1 - \sum_{k \neq j}\delta_{jk}\right]\right). \tag{8}$$

Using (2) and (6), equation (8) can then be rewritten as

$$E = \sum_{j=1}^{N} s_j \varepsilon_j \left(1 - R_j\right) \tag{9}$$

where $s_j = q_j/q$ denotes the quantity share of product j.

[10] This definition of group elasticity is based on total volume and produces an elasticity measure that generally is dependent on the units of measurement chosen for the volume of each product. In the Appendix, we provide alternative definitions based on revenues.

Equation (9) shows the general relationship between the group elasticity of demand, own-price elasticities, RRs and the quantity shares of the individual products. That is, the group elasticity of demand is equal to the weighted average of own-price elasticities multiplied by the respective products' diversion to outside goods. Intuitively, the group elasticity of demand goes to zero as RRs go to one. Indeed, when RRs go to one, changes in price merely lead to relative shifts in consumption between products, but there is no overall change in the level of demand, so $E = 0$. Conversely, when RRs go to zero, the group elasticity converges to the weighted average of own-price elasticities. Again, this is intuitive, because when RRs go to zero there is effectively no substitution (and hence competition) between inside products. For this reason, individual products' demands are no more elastic than aggregate demand in this case (but they would be with substitutability).

3. Symmetric Recapture Ratios

We will now explore in more detail the specific implications of equations (9) for the determination of RRs. This section begins by considering the natural benchmark of symmetric RRs, which applies when products are equally prone to losing demand to outside goods (or non-consumption) in response to a unilateral price increase.

If one assumes symmetric RRs (i.e., $R_j = R$ for all j), equation (9) implies

$$E = (1 - R)\sum_{j=1}^{N} s_j \varepsilon_j. \tag{10}$$

Assuming for simplicity that each product is sold by a different firm, profit maximization implies that in equilibrium the margin of each product is equal to:[11]

$$m_j = 1/\varepsilon_j \tag{11}$$

[11] In (11) and hereafter, we express all elasticities in absolute value and assume finite own-price elasticities so that $m_j > 0$ for all j.

Using (11) to substitute for ε_j in (10), we solve for R and obtain

$$R = 1 - \bar{M}E \tag{12}$$

where $\bar{M}$ denotes the volume-weighted harmonic average margin

$$\bar{M} = \frac{1}{\sum_{j=1}^{N} s_j/m_j}. \tag{13}$$

Equation (12) is a remarkably simple expression that only depends on the average margin and the group elasticity of demand. We summarize these results in the following proposition.

Proposition 1. *If recapture ratios are symmetric, i.e., $R_j = R$ for all j, then $R = 1 - \bar{M}E$, where $\bar{M}$ denotes the weighted harmonic mean of firms' profit margins and E denotes the group elasticity of demand (in absolute value).*

To illustrate the implications of Proposition 1, consider a set of N products with an average margin of 40% and a group elasticity of 0.75. Then, under the assumptions leading to Proposition 1, the recapture ratio is given by $1 - 0.4 \cdot 0.75 = 70\%$. Thus, following a unilateral price increase of one of the products, 70% of the sales lost by that product are captured by the other $N - 1$ products, whereas the remaining 30% are lost to outside goods.

4. Proportional Recapture Ratios

In many situations, symmetric RRs will be a plausible assumption.[12] In other situations, however, RRs may be asymmetric. In particular, this is true in the polar

[12] For example, when customers view differentiation among products as mostly horizontal rather than vertical, it may be natural to assume that diversion to outside goods does not depend on the particular product considered.

case when there is proportional diversion to outside goods (i.e., when the IIA assumption holds between inside and outside goods). In that case, RRs are given by the following expression:[13]

$$R_j = \frac{1-s_j}{1-s_j+s_0} \tag{14}$$

where s_0 denotes the "share" of outside goods (so that the size of the "market" of total potential sales is $1 + s_0$).[14] To see that (14) holds, note that the numerator $1 - s_j$ is the share of inside products other than product j from which diversion originates, while the denominator $1 - s_j + s_0$ is the total share of inside and outside goods, again excluding product j. Under the proportionality assumption, the recapture ratio must then be equal to the ratio of these two expressions.[15]

An immediate implication of (14) is that products with higher shares exhibit lower proportional RRs than products with lower shares (i.e., R_j is decreasing in s_j).

Since s_0 is an unknown variable, we again need to relate equation (14) to the group elasticity in order to be able to substitute for s_0 with observable parameters. From (9), (11) and (14), it follows that

$$E = \sum_j \frac{s_j}{m_j} \frac{s_0}{1+s_0-s_j}. \tag{15}$$

[13] The derivations that follow *solely* require the proportionality assumption to hold between inside products (with diversion R_j) and outside goods (with diversion $1 - R_j$). That is, proportionality is *not* required to hold among inside products themselves.

[14] This attribution of a "share" to outside goods is common in models of competition that satisfy the IIA property (e.g., the antitrust logit model, PCAIDS or the CES demand system). As noted in footnote 3, shares may be calculated based on total customers or based on the subset of arriving or departing customers, or some other measure of diversion that adds up to 1 for inside products.

[15] Formally, proportionality implies that diversion to outside goods $(1 - R_j)$ is proportional to their share (s_0). Similarly, diversion to inside products (R_j) is proportional to their share $(1 - s_j)$. That is, $1 - R_j = \mu s_0$ and $R_j = \mu(1 - s_j)$, where μ is the proportionality factor. Solving these two equations for R_j and μ yields equation (14).

For a given set of parameter values, equation (15) cannot generally be solved in closed form for s_0. However, with a finite number of brands, its recursive formulation can be solved numerically in applications, and the numerical solution for s_0 can then be substituted back into (14) to obtain a solution for R_j. We can therefore state the following result.

Proposition 2. *If recapture ratios are proportional to the share of inside and outside goods, then* $R_j = (1 - s_j)/[1 - s_j + s_0(\cdot)]$ *for all* j*, where* $s_0(\cdot) \geq 0$ *is the unique solution of the equation* $E = \sum_j s_j s_0/[m_j(1 + s_0 - s_j)]$*.*

Proof. We need to show that (15) has a unique solution $s_0 \geq 0$.

(i) If $E = 0$, then $s_0 = 0$ is the unique solution of (15) that satisfies $s_0 \geq 0$.

(ii) If $E > 0$, then the right-hand side of (15) is smaller than E at $s_0 = 0$ and greater than E if $s_0 \to \infty$ since it converges to $\sum_j s_j \varepsilon_j > E$. The latter inequality follows because the average own-price elasticity must be greater (in absolute value) than the group elasticity as the products are substitutes (see equation (9)). As $m_j > 0$ and $0 < s_j < 1$, the right-hand side of (15) is continuous and strictly increasing in s_0 for all $s_0 \geq 0$. Hence, it has a unique solution $s_0 > 0$. ■

It is instructive to compare how estimated RRs differ depending on whether the assumption of symmetric RRs (Proposition 1) or the assumption of proportional RRs (Proposition 2) is made. Denoting the former by R^S (for "symmetric" RR) and the latter by R_j^P (for "proportional" RRs), we obtain the following result.

Proposition 3. *(i) When $E = 0$ or when shares are symmetric (i.e., $s_j = 1/N$ for all j), then proportional RRs are equal to the symmetric RR, i.e., $R_j^P = R^S = 1 - \bar{M}E$ for all $j = 1, \dots, N$. (ii) When $E > 0$ and shares are asymmetric, $R_j^P < R^S$ for products with high shares and $R_j^P > R^S$ for products with low shares.*

Proof. (i) If $E = 0$, then (10) implies $R^S = 1$, and (15) implies $s_0 = 0$ so that (14) implies $R_j^P = 1$. If $s_j = 1/N$ for all j, then solving (15) for s_0 yields

$$s_0 = \frac{N-1}{N} \frac{E}{\sum_j \frac{1}{m_j} - E}. \tag{16}$$

Substituting this expression and $s_j = 1/N$ into (14), using (13) and rearranging then yields $R_j^P = 1 - \bar{M}E$. Thus, in both cases, $R_j^P = R^S = 1 - \bar{M}E$, as claimed.

(ii) From (12) and (14), $R_j^P < R^S$ if and only if

$$\frac{1-s_j}{1-s_j+s_0} < 1 - \bar{M}E. \tag{17}$$

Solving this for s_j yields

$$s_j > 1 - \frac{1-\bar{M}E}{\bar{M}E} s_0. \tag{18}$$

Hence, $R_j^P < R^S$ if and only if s_j is sufficiently large. What remains to show is that (18) must hold for the product with the highest share $\bar{s}$ and must fail to hold for the product with the lowest share $\underline{s}$. To do so, substitute for $\bar{M}$ and E from (13) and (15) into (18) and rearrange, which yields

$$\sum_k \frac{s_k}{m_k} \frac{s_0(s_k - s_j)}{(1+s_0-s_k)(1+s_0-s_k)} < 0. \tag{19}$$

Inequality (19) must hold for $s_j = \bar{s}$ since by the definition of $\bar{s}$, we must have $s_k - \bar{s} \leq 0$ for all k, with $s_k - \bar{s} < 0$ for some k. Similarly, the inequality sign in (19) must be reversed for $s_j = \underline{s}$. ■

Proposition 3 shows that when firms' shares are symmetric or when the group elasticity of demand is zero, the assumption of symmetric RRs and the assumption of proportional RRs generate identical results. However, in most real-world merger cases, products have asymmetric shares and the group elasticity of demand is not equal to zero. Proposition 3 shows that proportional RRs will then be smaller (larger) than the symmetric RR for products with relatively higher (lower) shares.[16]

5. Vertical Product Differentiation

A second important case where RRs are asymmetric is when there is significant vertical product differentiation. For instance, in response to a unilateral price increase, purchasers of luxury cars likely would be much less inclined to switch to public transportation (the outside good) than purchasers of regular cars. In other words, the RRs of luxury cars are likely to be significantly larger than the RRs of regular cars. In this section, we therefore consider the case of vertically differentiated products with two nests: a regular segment with n brands (brands $1, \dots, n$) and a premium segment with $N - n$ brands (brands $n + 1, \dots, N$).[17]

Let $\hat{s} = \sum_{j=1}^{n} s_j \in (0,1)$ denote the aggregate share of regular brands. The aggregate share of premium brands is thus given by $1 - \hat{s}$. Similarly, let $\widehat{M} = \left(\sum_{j=1}^{n} s_j/(\hat{s} m_j)\right)^{-1}$ denote the weighted harmonic mean of regular brands' margins.

[16] This implies that the closeness of competition between merging products with high (low) market share is smaller (larger). Thus, all else equal, proportional RRs lead to less (more) pronounced merger effects than symmetric RRs when the merging firms' market shares are high (low). In that sense, the assumption of proportional RRs is more (less) favorable for merging parties with high (low) market shares than the assumption of symmetric RRs.

[17] As in the previous sections, the analysis also can be undertaken using revenue-based variables as defined in the Appendix. Again, the N brands need not necessarily constitute a relevant antitrust market. Indeed, the existence of significant vertical differentiation suggests that antitrust markets may in fact be narrower than the overall set of goods (e.g., delineated along the boundaries of the two nests).

To be concrete, we consider the case where RRs are asymmetric between nests but symmetric within nests. Specifically, we assume that $R_j = R < 1$ for $j = 1, \dots, n$ (regular brands) and $R_j = 1$ for $j = n + 1, \dots, N$ (premium brands). That is, purchasers of premium brands may purchase another premium brand or "purchase down" to a regular brand in response to a unilateral price-increase of their preferred option, but will not switch to an outside good. Thus, the recapture ratio for a premium brand is equal to 1. Purchasers of regular brands, however, may purchase another regular brand, “purchase up” to a premium brand or switch to an outside good if the price of their preferred choice increases unilaterally. Thus, the recapture ratio for a regular brand is smaller than 1.

Since $R_j = 1$ for premium brands, equation (9) becomes

$$E = \sum_{j=1}^{n} s_j \varepsilon_j (1 - R_j). \tag{20}$$

Using equation (11) and symmetry among regular brand RRs, we thus have

$$E = (1 - R) \sum_{j=1}^{n} \frac{s_j}{m_j}. \tag{21}$$

Dividing this equation by $\hat{s}$ on both sides, solving for R and rearranging then yields the following result.

Proposition 4. *In the model with vertical product differentiation, the recapture ratio for premium brands is equal to 1 and, for regular brands, it is given by* $R = 1 - \widehat{M}E/\hat{s}$, *where* $\widehat{M}$ *denotes the weighted harmonic mean of regular brands' profit margins,* $\hat{s}$ *denotes the aggregate share of regular brands and* E *denotes the group elasticity of demand.*

Again, we can compare this case with asymmetric RRs and the benchmark case in Proposition 1 where the RR of every brand is R^S. The RR of a premium brand is assumed to be equal to 1 and thus is higher than R^S. Denoting the RR of a regular brand given in Proposition 4 by R^V (for "vertical" RR), we obtain the following result.

Proposition 5. *All else equal, vertical RRs for regular brands are smaller than symmetric RRs ($R^V < R^S$).*

Proof. Propositions 1 and 4 imply that $R^V < R^S$ if and only if $\widehat{M}/\hat{s} > \bar{M}$. From the definitions of $\widehat{M}$ and $\bar{M}$,

$$\frac{\widehat{M}}{\hat{s}} = \frac{1}{\sum_{j=1}^{n} s_j/m_j} \quad \text{and} \quad \bar{M} = \frac{1}{\sum_{j=1}^{n} s_j/m_j + \sum_{k=n+1}^{N} s_k/m_k}. \tag{22}$$

Thus, $\widehat{M}/\hat{s} > \bar{M}$ if and only if

$$\frac{1}{\sum_{j=1}^{n} s_j/m_j} > \frac{1}{\sum_{j=1}^{n} s_j/m_j + \sum_{k=n+1}^{N} s_k/m_k}. \tag{23}$$

This inequality must hold, since $\sum_{k=n+1}^{N} s_k/m_k > 0$. ■

Compared to symmetric RRs, RRs when products are vertically differentiated are lower for regular brands and higher for premium brands, as one would expect.

6. Application to Competitive Effects in Merger Control

The previous results can readily be applied to competitive effects analyses in merger control and other areas of antitrust where diversion ratios play a key role. In particular, the diversion ratio from product i to product j is given by the following expression when the proportionality assumption holds among inside products (i.e., when the products under consideration are equally close substitutes to each other):[18]

[18] When goods are asymmetrically positioned in terms of closeness of substitution, equation (24) becomes $\delta_{ij} = \sigma_{ij} R_i$, where σ_{ij} denotes product j's share of diversion away from product i within the group of inside products. As noted earlier, estimates of such shares of diversion may become available at the later stages of an investigation, since many firms track the identity of customer gains and losses from and to rivals.

$$\delta_{ij} = \frac{s_j}{1-s_i} R_i \tag{24}$$

From (26) it is clear that individual diversion ratios δ_{ij} cannot be meaningfully quantified without sensible estimates of RRs δ_i. Propositions 1, 2 and 4 above can then be readily applied to obtain such estimates.

When it is sensible to assume symmetric RRs (i.e., when there is no reason to assume that some inside products have appreciably different diversion to outside goods than others), Proposition 1 implies that (24) can be expressed as follows:

$$\delta_{ij} = \frac{s_j}{1-s_i}(1 - \overline{m}\varepsilon) \tag{25}$$

This formulation of diversion ratios solely depends on the products' shares, margins and group elasticity of demand, which often can be approximated with a reasonable degree of accuracy even at the early stages of an antitrust investigation. Note, in particular, that (25) is independent of cross-price elasticities and more complex curvature properties of demand functions, whose estimation would require the availability of much richer data sets and the use of advanced econometric techniques.

Similarly straightforward diversion metrics can be derived in the case of proportional diversion between inside and outside goods (in which case RRs are asymmetric). In this case, Proposition 2 implies that (24) can be expressed as follows:

$$\delta_{ij} = \frac{s_j}{1-s_i+s_0(\cdot)} \tag{26}$$

where $s_0(\cdot)$ is defined implicitly by equation (15). This expression depends on the same simple parameters as equation (25).

In the case of vertically related nests analyzed in Section 5, Proposition 4 implies that (24) can be expressed as follows:

$$\delta_{ij} = \frac{s_j}{1-s_i}\left(1 - \frac{\widehat{m}}{\hat{s}}\varepsilon\right) \tag{27}$$

In applied analyses, the diversion ratio approximations in equations (25) to (27) can be used in two ways. First, at the early stages of a merger investigation, they permit using a variety of quantitative tools to evaluate the potential competitive effects of the merger, requiring only a relatively small amount of information. Second, at more advanced stages of a merger proceeding, data on the patterns of substitution from one product to another often become available (e.g., in the form of customer switching data). It is then possible to replace the products' shares in equations (25) to (27) with more accurate indicators of diversion between inside products. This addresses the potential shortcoming of the proportionality assumption and produces more reliable estimates of merger effects by taking proper account of differentiated patterns of closeness of substitution.

7. CONCLUSION

In this paper, we have analyzed the general relationship between *recapture ratios* and the group elasticity of demand. This has allowed us to derive simple solutions for estimating recapture ratios in merger control proceedings, both for the case of symmetric recapture ratios and for the case of asymmetric recapture ratios. For the case of asymmetric RRs, we have considered the natural benchmarks of proportional diversion and vertical product differentiation.

The results allow us to approximate diversion ratios between competing products on the basis of simple, observable variables such as the products' margins, shares, and group elasticity of demand. In applied work, this is useful since most quantitative tools for assessing the competitive effects of mergers heavily rely on diversion ratios.

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APPENDIX

This appendix provides alternative definitions and formulas that are useful in situations where firms sell multiple products or when there is no natural way to choose units of measurement and aggregate different products. In particular, one might prefer to use total revenue (or total spending) instead of total volume:

$$S = \sum_{k=1}^{N} p_k D_k(p_1, \dots, p_N) \tag{3*}$$

and define the group elasticity of demand as the elasticity of total revenue minus one:

$$E^* = \frac{t}{S}\frac{d}{dt}\sum_{k=1}^{N} tp_k D_k(tp_1, \dots, tp_N)\,|_{t=1} - 1 = \sum_{k=1}^{N}\sum_{j=1}^{N} p_k \frac{\partial D_k}{\partial p_j}\frac{p_j}{S}. \tag{4*}$$

For the diversion ratio from product j to product k, one might prefer to use

$$\delta_{jk}^* = -\frac{p_k}{p_j}\frac{\partial D_k/\partial p_j}{\partial D_j/\partial p_j} \tag{5*}$$

and for product j's recapture ratio one might prefer to use

$$R_j^* = \sum_{k \neq j} \delta_{jk}^*. \tag{6*}$$

Denoting the revenue share of product j by $s_j^* = p_j\, q_j/S$, we have

$$E^* = \sum_{j=1}^{N} s_j^* \varepsilon_j \left(1 - R_j^*\right) \tag{9*}$$

In the case of symmetric RRs, we have

$$R^* = 1 - \bar{M}^* E^* \tag{12*}$$

where $\bar{M}^*$ denotes the revenue-weighted harmonic average margin

$$\bar{M}^* = \frac{1}{\sum_{j=1}^{N} s_j^*/m_j}. \tag{13*}$$

In the case of proportional RRs, we have

$$R_j^* = \frac{1-s_j^*}{1-s_j^*+s_0^*}. \tag{14*}$$

where s_0^* is the solution of

$$E^* = \sum_j \frac{s_j^*}{m_j}\frac{s_0^*}{1+s_0^*-s_j^*}. \tag{15*}$$