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Abstract

The fixing of benchmark rates such as Libor, Euribor and FX has proven vulnerable to manipulation. Individual rate-setters may have incentives to fraudulently distort their submissions. For the contributing banks to collectively agree on the direction in which to rig the rate, however, their interests need to be sufficiently aligned. In this paper we show how a continuous benchmark rates cartel could be sustained by preemptive portfolio changes. Exchange of information facilitates front running that allows members to reduce conflicts in their trading books. Designated banks then engage in eligible transactions rigging to justify their submissions. As the cartel is not able to always find stable cooperative submissions against occasional extreme exposure values, there is episodic recourse to non-cooperative quoting. The benchmarks remain vulnerable to these cartel mechanisms, also after the implementation of recent and proposed reforms. It is not obvious how to make them more resilient to collusion. Periods of heightened volatility in the rates can be indicative of collusion.

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Trader RBS: *"It's just amazing how Libor fixing can make you that much money or lose if opposite. It's a cartel now in London."*

Trader Deutsche Bank: *"Must be damn difficult to trade man, especially if you are not in the loop."*[1]

1 Introduction

Financial benchmark rates such as Libor, Euribor and FX globally underlie large values in economic transactions. The Libor and Euribor are calculated daily for various maturities and currencies as the trimmed average of submissions from a number of panel banks to reflect their capacity to borrow unsecured funds on the London and European interbank market. Between 370 trillion and 530 trillion dollar worth of interest rate derivatives, consumer and commercial credit—or between 4.5 and 6.5 times global GDP—are estimated to directly derive their value from these rates.[2] The exponential rise of the over-the-counter (OTC) derivatives markets in the late 1980s through early 2000s significantly increased the volumes traded on the benchmarks. The Libors and Euribors are key variables in portfolio and risk management decisions and barometers of financial sector health.

Foreign exchange (FX or forex) rates are prices in the currency market, the largest market in the world, with transactions taking place around the clock. World Market/Reuters (WMR) provides the most widely used standardized forex benchmarks, including the 4 p.m. London close. They are determined as the median value of buy and sell transactions executed by forex traders, primarily large banks, for clients as well as their own accounts during specific short time windows. Trading in the foreign exchange market is worth in the trillions of dollars a day.[3] The forex WMR rates allow fund managers to value holdings and are used in forwards and other multi-currency contracts.

The fixing of these financial benchmarks has proven vulnerable to manipulation. The rates are determined on the basis of contributions by market participants who also trade in the financial products that are valued on the benchmarks, giving them financial incentives to manipulate their contributions resulting from their trading exposure positions. Suspicion of manipulation of the benchmark rates arose when the *Wall Street Journal* reported that in the gathering of the global financial crisis

[1] Transcript of conversations on 19 August 2007, submitted as evidence in *Tan Chi Min v The Royal Bank of Scotland*, S939/2011, Singapore High Court, as quoted in "RBS Instant Messages Show Libor Rates Skewed for Traders," *Bloomberg*, 26 September 2012.

[2] HM Treasury, "The Wheatley Review of Libor: Final Report," August 2012, page 3 and Financial Stability Board, "Reforming Major Interest Rate Benchmarks," July 2014, page 6.

[3] The Bank of International Settlements, "Triennial Central Bank Survey: Foreign Exchange Turnover in April 2016," September 2016, on page 3 reports an average daily trading volume in April 2016 of $5.1 trillion, down from $5.4 trillion in April 2013.

the Libors appeared to diverge periodically from other proxies of bank borrowing costs and risk, in particular credit default swaps (CDS) spreads.[4]

A few banks, including Barclays, admitted early on to misreporting, and numerous major banks in the panels have been prosecuted since. As the individual submissions of banks used to be published together with the calculated rates for all to see, panel members could try to come across more creditworthy than they actually were by underreporting their true borrowing costs to the Libor, and so avoid inducing counterparties to charge them higher risk premiums for suspicion of liquidity problems—so-called 'low-balling'. In some instances, the practice was endorsed by senior management—as a senior treasury manager at Barclays instructed a submitter:

> "[S]tick within the bounds[,] so no head above [the] parapet."[5]

Vaughan and Finch (2017) even suggest that the Bank of England permitted, if not told, the British panel banks to low-ball Libor and break the code of conduct to maintain financial stability.[6]

Subsequent investigations into Libor and Euribor by government authorities around the world uncovered evidence from emails, phone recordings and instant messages of panel bank employees manipulating submissions on a large scale, as well as colluding to increase trading profits already before the crisis.[7] *Bloomberg* reporters discovered that also the foreign exchange rates were systematically rigged by colluding traders. Many of them later turned out to have worked for the same large banks involved in the Libor and Euribor manipulations as well.[8]

In by invitation only chatrooms with names such as 'The Cartel' and 'The Mafia', senior currency traders met on a regular basis to agree on their strategies.[9] These

[4] C. Mollenkamp and M. Whitehouse, "Study Casts Doubt on Key Rate; WSJ Suggests Banks may have Reported Flawed Interest Rate Data for Libor," *Wall Street Journal*, 29 May 2008.

[5] Commodity Futures Trading Commission, "Order Instituting Proceedings: Barclays Bank PLC," 27 June 2012, page 21, recorded 30 November 2007.

[6] Vaughan and Finch (2017), Chapter 11 details contacts between Barclay's CEO Bob Diamond and Paul Tucker, deputy governor of the Bank of England. During a call on 29 October 2008, Diamond internally reported: "Mr Tucker states the level of calls he was receiving from Whitehall were 'senior' and that while he was certain we did not need advice, that it did not always need to be the case that we appeared as high as we have recently." (*Op.cit.*, page 97).

[7] See for example U.S. Department of Justice, "Statement of Facts, Non-prosecution Agreement: Barclays Bank PLC," 26 June 2012, Commodity Futures Trading Commission, "Order Instituting Proceedings: Barclays Bank PLC," 27 June 2012; and Financial Services Authority, "Final Notice: Barclays Bank PLC," 27 June 2012. Similar documents of DoJ, CFTC and FSA cases exist for Deutsche Bank, Lloyds, Rabobank, RBS and UBS. See also European Commission, "Antitrust: Commission fines banks €1.49 billion for participating in cartels in the interest rate derivatives industry," 4 December 2013.

[8] L. Vaughan, G. Finch and A. Choudhury, "Traders Said to Rig Currency Rates to Profit off Clients," *Bloomberg*, 12 June 2013.

[9] Commodity Futures Trading Commission, "Examples of Misconduct in Private Chat Rooms," 12 November 2014; Vaughan and Finch (2017), page 174.

included front running client orders and pushing through trades in the 60-second window when the benchmarks were set, so-called 'banging-the-close', by which forex traders could sell (buy) their own currency holdings at a high (low) price while pushing down (up) the rate at which to later buy (sell) currency from (to) clients. They would also withhold bids or offers for certain currencies in order to avoid moving the exchange rates in directions adverse to open positions by the cartel members, and place fake orders for currencies that were not intended to be actually executed, seeking to push rates prior to the closing period—so-called 'painting the screen'—and 'wash trades' to pocket broker commissions.[10]

Despite evidence of wider coordination in the benchmark rates fixings, most investigations so far have focussed on fraudulent misreporting in breach of the banking code of conduct and client confidentiality by individual traders trying to influence rate-setters within a bank, sometimes bilaterally or between a few banks during the financial crisis. The cases were handled for misconduct threatening the integrity of the Libor and Euribor, in the US by the Department of Justice's Criminal Division's Fraud Section and the Commodity Futures Trading Commission (CFTC), and in the UK by the Financial Service Authority (FSA)—the later Financial Conduct Authority (FCA). Certain traders would on occasion have communicated with others in the panel, which portrays the cases as incidents of favors done between rogue traders for their own benefit, possibly against the interests of their employers.

Antitrust cases in financial benchmark setting are few so far. While the DoJ's Antitrust Division was involved in the fraud investigations, it did not prosecute for collusion. Initially, in a private antitrust damages action, the Federal Court of New York ruled that the Sherman Act would not apply to the Libor setting mechanism, as it was deemed a cooperative rather than competitive process.[11] This ruling got overturned on appeal, however, by the United States Court of Appeals for the Second Circuit in Manhattan, which determined that Libor manipulation did constitute price-fixing as a per se antitrust violation under Section 1.[12]

European cartel investigations into Libor and Euribor started with leniency applications by UBS, RBS and Barclays. Moreover, in a hybrid settlement and subsequent decisions, the European Commission established an Article 101 TFEU cartel violation, in euro, Japanese yen and Swiss franc interest rate derivatives against nine of the largest panel banks and two brokers that had facilitated the cartel for record fines.[13]

[10] Scopino (2016), page 613.

[11] *In re: LIBOR-Based Financial Instruments Antitrust Litigation*, No. 935 F. Supp. 2d 666, 29 March 2013.

[12] *In re: LIBOR-Based Financial Instruments Antitrust Litigation*, No. 13-3565, 2nd Circuit, 23 May 2016.

[13] European Commission, *Case A.39914—Euro Interest Rate Derivatives* and European Commission, *Case AT.39861—Yen Interest Rate Derivatives*, a hybrid settlement with Barclays, Deutsche Bank, Société Générale, RBS, UBS, JP Morgan, Citigroup and RP Martin (broker) on 4 December 2013 and later infringement decisions for *YIRD* against broker ICAP on 4 February 2015 and for *EIRD* against Crédit Agricole, HSBC and JPMorgan Chase on 7 December 2016; European Com-

In *EIRD*, collusion was established to have started already in September 2005.

Forex market prosecutions by UK and US authorities have been concluded with settlements as well.[14] The European Commission has not disclosed anything to date about its ongoing cartel investigations into the foreign exchange market.[15] As a result, little or no information about the supposed inner workings of the benchmark rates cartels is public.

Commentators have argued that the benchmark systems would be too complex to conspire against, due to the size of the market, misaligned interests and the trimming mechanism in the rate setting processes.[16] It is indeed not obvious that a for-profit cartel in the fixing of the benchmark rates could have worked. Contrary to conventional cartels, in which all members typically want to increase product prices, often the interests of the panel banks in manipulating the benchmark fixings will not be aligned, as their exposures to the rates fluctuate partly unpredictably over time. Different banks will regularly find themselves on opposing sides of the market, where some gain from an increase in one or more of the rates, while others benefit from a decrease. Such diverse and constantly changing payoff incentives are a challenge to cartel stability. The trimming of the higher and lower Libor and Euribor submissions adds to the difficulty. In addition, the proposed reforms are aimed at basing the interbank rates partly on actual transactions. Manipulation often requires suboptimal transactions against other than the going rates in order to move them, so that collusion can be costly.

In this paper we show how a cartel in the fixing of benchmark rates can work,

mission, *Case AT.39924—Swiss Franc Interest Rate Derivatives*, two settlement decisions on 21 October 2014, one with RBS and JP Morgan on derivatives based on the Swiss franc Libor and one with RBS, UBS, JP Morgan and Crédit Suisse for bid-ask spreads charged on Swiss Franc interest rate derivatives.

[14] See for example U.S. Department of Justice, "United States of America v. Citicorp, Plea Agreement," 20 May 2015; Commodity Futures Trading Commission, "Order Instituting Proceedings: Citibank NA," 12 November 2014 and Financial Conduct Authority, "Final Notice: Citibank NA," 11 November 2014. Similar documents of DoJ, CFTC and FCA cases exist for JPMorgan Chase, RBS, UBS, HSCB and Barclays.

[15] European Commission, "Statement on the euro interest rate derivatives case," 20 May 2014; "FX Probe Said to Emerge From Shadows as EU Seeks Bank Data," *Bloomberg*, 3 June 2016.

[16] Euribor-EBF officials believed that the composition of its bank panel made it impossible for a cartel to fix the index. Director Cedric Quéméner claimed that too many banks, certainly more than the 9 investigated, would need to be involved for manipulation of Euribor to be possible. Source: "Euribor cuts Libor Adrift in Scandal Storm," *EurActiv*, 26 July 2012, in which an industry source is quoted to have added: "[I]f these banks were to convince each other that they want the same rate level, they would need to find at least another 15 different banks with the same interest as their own." University of Florida professor of finance Andy Naranjo, for example, was skeptical of the ability of traders to manipulate foreign exchange rates due to its large size. See L. Vaughan, G. Finch and A. Choudhury, "Traders Said to Rig Currency Rates to Profit off Clients," *Bloomberg*, 12 June 2013. Scopino (2016) at page 639 does recognize collusive schemes in the financial benchmark manipulations and argues that the CFTC should be given broad authority to combat such anticompetitive conduct.

despite conflicting and time-varying interests and the trimming. We develop a model in which panel banks exchange information and use it to maximize joint profits from their portfolio exposure positions by both agreeing on their contributions to the fixing and subsequent adaptations to their exposure positions. Two complementary mechanisms inspired by evidence of the fraud investigations make this possible. Inside information would allow colluding banks to engage in lucrative front running and adjust their trading books in a time window of opportunity. In addition, in order to distort the rates in the agreed direction that maximizes cartel joint profits, designated banks engage in eligible transactions rigging. These are suboptimal transactions of the type that is to justify a panel member's contributions to the rate calculation process, aimed at benefitting the bank's overall trading position.

OTC derivatives are particularly suited for front running, since they in large part determine a bank's exposure to the rates, are highly volatile short and long, and non-eligible for calculation of the benchmark.[17] We find that the optimal cartel strategy of continuous collusion with portfolio position-specific stable collusive actions is not generically attainable. Instead, when the cartel members observe that the occasional extreme exposure value gives one (or more) of them incentive to deviate, all revert temporarily to independent quoting to stabilize the cartel.

There have been some notable cases of collusion in banking and financial markets. Christie and Schultz (1994) uncovered in a study of pricing patterns that NASDAQ market makers avoided quoting prices in odd eights of a dollar, suggesting implicit collusion to maintain wide bid-ask spreads, that led the DoJ to intervene.[18] VISA, MasterCard and American Express have been prosecuted for restricting price competition and sued for antitrust damages in the US in various cases since the mid 1990s. The European Commission brought cartel cases also against large cooperatives of international banks in objection to the joint setting of multilateral interchange fees (MIFs) in determining credit card charges. In 2002, it fined eight Austrian banks participating in the 'Lombard Club' for fixing interest rates and service fees.[19]

The emerging literature on benchmark rates focuses almost exclusively on manipulation by one or a few rates-setters. Abrantes-Metz et al. (2012) see episodes of suspicious Libor submissions by individual banks in comparison to the federal fund effective rate and 1-month T-Bill rates, yet no evidence for widespread manipulation. Monticini and Thornton (2013) find more material anomalous patterns for the same period when using the relationship between Libor and large, unsecured certificate of deposit rates. Kuo et al. (2012) compare Libor quotes to bank bids in the Federal Reserve Term Auction Facility, deduced borrowing costs to find that Libor submissions were significantly lower than comparison rates during the crisis, which could indicate

[17] Financial Stability Board, "Market Participants Group on Reference Rate Reform, Final report," March 2014.

[18] *United States v. Alex. Brown & Sons, et. al (Nasdaq Market Makers)*, July 17, 1996.

[19] See European Commission, "Commission fines eight Austrian banks in 'Lombard Club' cartel case", 11 June 2002.

such low-balling. Gandhi et al. (2016) estimate monthly Libor-related positions and find a relation between the positions and banks' submissions, which is stronger for banks outside the US, where enforcement is historically weaker.

Snider and Youle (2012) study the incentives behind portfolio based manipulation in a Bayesian game of strategic Libor quote submission as signals of creditworthiness between individual banks that each maximize their own trading profits. Youle (2014) uses a model based on a non-cooperative game of incomplete information to estimate banks' exposures and finds evidence suggesting that Libor was downward biased during the recent crisis. Chen (2017) models the Libor mechanism as a Bayesian signaling game to find that banks' individual manipulations decrease with the panel size and number of quotes used in the calculation. The result of a distribution-free bias does not hold under collusion. Diehl (2013) models portfolio and reputation incentives and compares the performance of different aggregates, such as the mean and the median, under individual manipulation.

A few papers consider agreements between two or several panel members. Eisl et al. (2017) calculate how Libor misreporting by one or several banks together could have moved the average, but do not analyze incentives. Using a time-varying threshold regression model, Fouquau and Spieser (2015) argue that the breaks they find are not consistent with exogenous money market shocks, suggesting manipulation by small groups of panel banks that they propose to identify using a hierarchical clustering method.

Several papers on forex rigging consider collusion more explicitly, but offer no cartel theory. Evans (2016) examines trading patterns around the fixing-window in a model of competitive trading to find that price changes display volatility and serial correlation around the fix that is more consistent with collusive manipulation than competitive trading. Ito and Yamada (2015) and Michelsberger and Witte (2016) confirm these findings on the volatility and serial correlation of price changes around the fix. Saakvitne (2016) argues in a model with strategic dealers that banging-the-close follows naturally as a solution to individual dealers' optimal execution-problem.

While Abrantes-Metz (2012) suggests changes to reduce the risk of collusion in the interbank lending benchmark rates, this has not been the prime objective of the reforms. Duffie and Dworczak (2014) and Duffie and Stein (2015) propose reforms for both types of benchmark rates against manipulation, but not collusion. Coulter and Shapiro (2015) also focus on preventing unilateral manipulation in Libor and for that purpose develop a whistle-blower mechanism that gets banks to reveal their true borrowing costs.

The cartel model we develop extends classic cartel theory to specific features of benchmark rate-setting. Payoff shocks are observed and when a bank draws an extreme value position it has incentive to deviate, as during booms in Rotemberg and Saloner (1986). There is, however, no obvious strategy the cartel could fall back to in order to avoid defection and assure continuous collusion. Moreover, the trimming mechanism introduces discontinuities in the objective function, which makes that

a continuous collusion strategy is not generally attainable. Instead, for the period the cartel reverts back to non-cooperative submitting with inside information, as in Fershtman and Pakes (2000). 'Price wars' thus are episodic recourses to non-cooperative (possibly honest) quoting that is short run unprofitable, as in Green and Porter (1984), but an integral part of the collusive strategy, not punishment.

The remainder of this paper is organized as follows. Section 2 provides more detail on the benchmark rate manipulations and identifies elements for the cartel mechanism we suggest. In Section 3 the model is introduced and theoretical results on cartel stability are developed. In Section 4, simulation exercises give insight into collusive rate patterns. In Section 5, we discuss possible damages resulting from collusion in the benchmark rates. Section 6 concludes. The source code of a software that calculates optimal cartel strategies is given in the appendix.

2 Benchmark Rates Fixing

The Libor and Euribor interbank rates are produced with quotes from panel banks. Libor used to be compiled for 10 currencies and 15 maturities, ranging from overnight to 12 months, with panel sizes varying for different currencies between 6 to 18 banks. Barclays Bank, Deutsche Bank, Lloyds TSB Bank and The Royal Bank of Scotland have been in all currency panels. Euribor was calculated for 15 maturities by a panel of over 40 banks.[20] A large number of banks, including those who had been part of all Libor currency panels, were in both the Libor and Euribor panels.

On every trading day before a certain time in the morning, a panel of banks submits quotes for a number of maturities that are intended to reflect the rate at which they could borrow these funds on the interbank market. Before reforms in response to the recent scandals, the Libor quotes were a prediction of the submitters' own rates and the Euribors what each bank believed to be "the rate at which interbank term deposits are being offered within the EMU zone by one prime bank to another."[21]

Each rate is calculated as the trimmed average of the panel banks' submissions. In the setting of the Libor, the middle 50% of the quotes are averaged, for Euribor the middle 70%. The Libor quotes are to be submitted at 11.00 a.m. GMT and published at 11.55 a.m.. Euribor quotes are submitted at 10.45 a.m. and published at 11.00 a.m. CET. The individual submissions are disclosed as well, albeit for Libor with a 3 months embargo as of 2013.

The forex WMR rates are computed half-hourly for 22 currencies, and hourly for over 150.[22] The most important of them is the 'London 4 p.m. fix' or 'WMR Fix'

[20] Contrary to Libor, the Euro OverNight Index Average (EONIA) is not determined by panel submissions, but calculated by the European Central Bank on the basis of all overnight interbank assets created before the close of Real Time Gross Settlement systems at 6 p.m. CET.

[21] European Money Markets Institute, *Euribor Code of Conduct*, June 2016. On this point, the formulation has not changed from the original code.

[22] Thomson Reuters, *WM/Reuters FX Benchmarks: Sport & Forward Rates Methodology Guide*,

which is calculated as the median value of a subset of foreign exchange transactions that occur during a short time window at 4 p.m. CET—of 1 minute around before the reforms and 5 minutes since. Another major daily fix is the 1.15 p.m. CET European Central Bank fix produced in a similar way by the ECB. The forex benchmarks are meant to be a snapshot of going rates.

WMR collects bid and order rates from actual trades every second during the fixing-window on the three highly liquid trading platforms Thomson Reuters Matching, EBS and Currenex. Trading occurs every millisecond and therefore only a sample of trades is captured. Valid trades are pooled together and their rates used for the fix. WMR further has the discretion to exclude trades if it deems them non-representative, before publishing the rates. Even though the method of calculation takes no account of trading volume, the larger banks are likely to have a big impact on the rate. In 2016, eight banks traded close to 60% of total trading volume in the market.[23]

2.1 Mechanisms for Collusion

Financial markets are highly transparent and market participants monitor each other closely. The designs of the interbank benchmark rates make them particularly vulnerable to cooperative manipulation. The Libor and Euribor panels consisted for long periods of time of a fixed number of known members, leading financial institutions that would have been in contact with each other and share information on various platforms and in different circles. The major traders on the foreign exchange market are also commonly known. Various investigations revealed evidence of mechanisms at work in the benchmark manipulation cases that may also have induced, facilitated, or be indicative of widespread collusion.

2.1.1 Coordination

The FSA concluded from its fraud investigation that Barclays had "acted in concert with other banks."[24] Traders texted messages such as this one, from a Barclays trader to a Deutsche Bank trader on Euribor: "[T]oday we need a low 3 month fixing, could you tell your guys as well if it suits you."[25] Or a Barclays trader telling a trader of another panel bank after they had coordinated on their Euribor submissions: "[T]his is the way you pull off deals like this (...) the trick is you must not do this

January 2017.

[23] "Citi tops Euromoney global FX poll again, but big banks lose grip," *Reuters*, 25 May 2016.

[24] Financial Services Authority, "Final Notice: Barclays Bank PLC," 27 June 2012, recital 11. Vaughan and Finch (2017) narrate that to Tucker asking why Barclays submitted such higher Libors than other panel banks, Diamond had responded that it was: "[B]ecause it was the only bank being even vaguely honest about its borrowing costs." (*Op.cit.*, page 96).

[25] Commodity Futures Trading Commission, "Examples of Misconduct from Written Communication," 23 April 2015, page 4, recorded 29 December 2006.

alone."[26] Many of the panel banks were involved in the manipulations and coordinated potentially between all of them. The U.S. Department of Justice (DoJ) had identified at least five banks involved when it concluded:

> "Barclays Euro swaps traders communicated with swaps traders at other financial institutions that were members of the EURIBOR Contributor Panel about requesting favorable EURIBOR submissions from the EURIBOR submitters at their respective banks."

CFTC officials found that "Everybody was false reporting" and that:

> "Libor was routinely being gamed by the banks that set it."[27]

A broker from ICAP was nicknamed 'Lord Libor' for sending a daily email with Libor predictions at 7 a.m. to more than a hundred traders and brokers, including representatives of almost all of the Libor panel banks.[28] The broker proved sensitive to manipulation requests by traders and ICAP was implied in the European Commission cartel case. In forex chat groups, the main currency traders in the market discussed rates, trading positions and intentions on a regular basis.[29]

Abrantes-Metz et al. (2012) interpret certain patterns in the Libor submissions as indicative of coordination. Figure 1 displays the intraday cross-sectional coefficient of variation in quotes, which is the standard deviation divided by the mean of the Libor quotes on a certain trading day. The authors finds it suspicious of collusion that the panel submits much more similar quotes before August 2007 than after, but do not find that the Libor is significantly different from its predicted level when comparing it with other measures of bank borrowing costs.

Arguably, manipulation other than by mass collusion was difficult, because of the way the Libor and Euribor are calculated. By discarding the top and bottom parts of the quotes, individual outliers would be discouraged.[30] Even though misreporting by one bank alone could have moved the average, the effect on the rate of a group of colluding banks theoretically becomes unbounded once the group consists of strictly more banks than the number of submissions that are trimmed from each side.[31] In

[26] U.S. Department of Justice, "Statement of Facts, Non-prosecution agreement: Barclays Bank PLC," 26 June 2012, page 13.

[27] CFTC lawyer Steve Obie and head of enforcement Greg Mocek quoted in Vaughan and Finch (2017), pages 42 and 76 respectively.

[28] Vaughan and Finch (2017), page 29.

[29] Commodity Futures Trading Commission, "Examples of Misconduct in Private Chat Rooms," 12 November 2014; Vaughan and Finch (2017), page 174.

[30] See Vaughan and Finch (2017), pages 17-18.

[31] See "Libor Flaws Allowed Banks to Rig Rates Without Conspiracy," *Bloomberg*, 16 July 2012; Eisl et al. (2017).

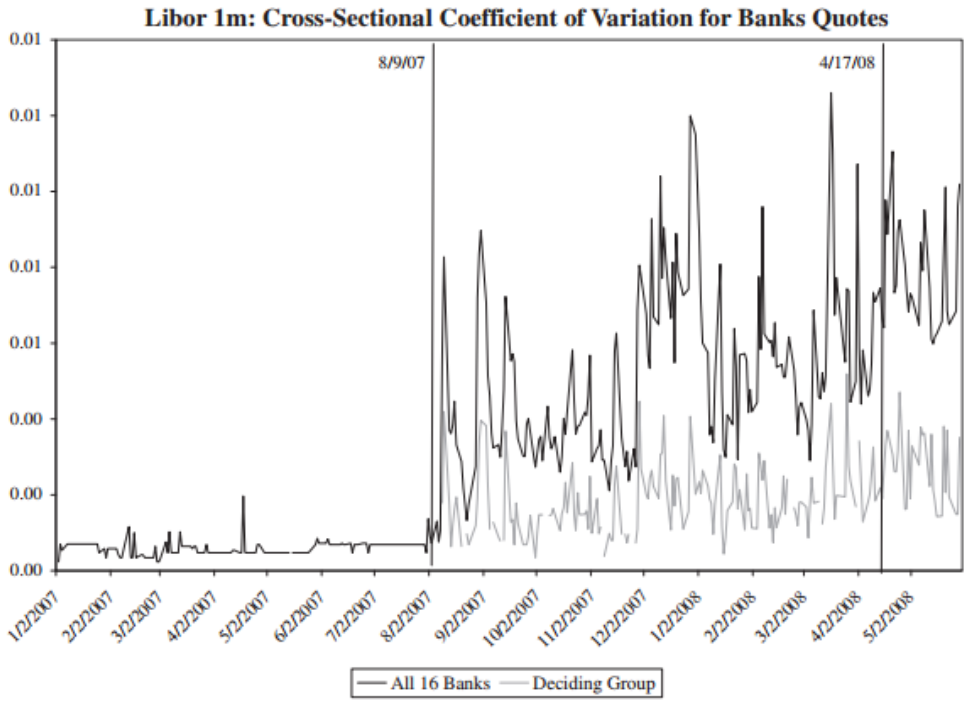

Figure 1: The 1-month Libor cross-sectional coefficient of variation for banks quotes in Abrantes-Metz et al. (2012).

the case of Libor, of a panel of 16, the top and bottom 4 were trimmed, so that a minimum of 5 banks would be needed to have an unbounded effect on the rate. For a Euribor panel of 44 banks, the top and bottom 6 or 7 would be trimmed, so 7 or 8 would do. In practice, the members would probably not submit quotes that are too extreme, as that might have drawn attention from regulators. Larger groups of colluding banks can have a smoother and much larger influence on the final rate than individually manipulating banks. Likewise, in the case of the forex' median value, it would have been difficult to affect the rates without pushing many transactions through in cooperation, because of the large size of the currency market.[32]

2.1.2 Episodic Break-up

Whereas coordination of the rates was pervasive, it appears not to have been continuous. For example does the European Commission in *EIRD* stress how:

> "*On occasion*, certain traders employed by [...] communicated and/or received preferences for an unchanged, low or high fixing of certain Euribor

[32] See Vaughan et al. (2013) and "How the Forex Scandal Happened," *BBC News*, 20 May 2015.

tenors. Those preferences depended on their trading positions/exposures."[33]

Attempts to coordinate the rates were not always successful, for example when a coconspirator was unable to accommodate another's trading position. For such events, the bank employees involved made an effort to explain themselves, apparently in order not to damage the cooperative relationship. For example does a trader apologize for mistakenly failing to accommodate a manipulation request from a submitter at another bank:

> "just b4 you beat me up.. .. I was in meeting so didn't do me libors today...thk they put .52 for Is..."

to which the submitter answered:

> "hahah no thats fine - thats what i set too cheers skip."[34]

There would be understanding for inability to comply with requests if it would create a significant conflict with a bank's portfolio position. On another occasion involving Rabobank, a submitter was asked to submit 3-month Euribor "at the ceiling" because, as another bank's swaps trader explained "I am long in fixings against Dec futures it cost me a fortune yesterday," upon which he was informed that Rabobank couldn't because:

> "long swaps need it low."[35]

Such incidents of independent quoting happened apparently only episodically. For example, on March 22, 2007, the Yen Libor submitter of Rabobank emailed the Lloyds TSB Yen submitter, requesting a "high lmthjpy libor set tomorrow please". The Lloyds submitter forwarded the email to two colleagues who were making the Yen Libor submissions for Lloyds TSB in his absence and explained:

> "We usually try and help each other out. .. but only if it suits .. .!"[36]

There was recognition that even though coordination would not be possible every period, the longer term collusive arrangement was valid and valuable. One submitter preemptively contacted a trader at another bank on March 28, 2008 to excuse that he could not follow in manipulation of the rate that day:

[33] European Commission, *Case A.39914—Euro Interest Rate Derivatives*, 4 December 2013, recital 32, emphasis added. The recital lists seven of such occasions. All bank names have been redacted.

[34] U.S. Department of Justice, "Statement of Facts, Deferred Prosecution Agreement: Rabobank," 29 October 2013, page 33, recorded January 5, 2007.

[35] U.S. Department of Justice, "Statement of Facts, Deferred Prosecution Agreement: Rabobank," 29 October 2013, page 37.

[36] Commodity Futures Trading Commission, "Order Instituting Proceedings: Lloyds Banking Group PLC," 28 July 2014, page 12.

> "Submitter-4: 'morning skip - [Trader-5] has asked me to set high libors today - gave me levels of lm 82, 3m 94....6m 1.02.'
>
> Trader-B: 'sry mate cant oblige today...i need em lower!!!'
>
> Submitter-4: 'yes was told by [a third party].. .just thought i'd let you know why mine will be higher .. .and you don't get cross with me."'[37]

The evidence suggests that longer term collusive arrangements were maintained, yet not continuously consummated. Jointly agreeing episodically to break-up coordination of submissions when the circumstances were not quite right appears to have been part of an ongoing collusive fix.

2.1.3 For-Profit

Manipulation aimed at enhancing trading results, depending on their trading positions, sometimes accepting current short-run losses for long run future gains.[38] A bank with a net lending position would profit from a higher Libor or Euribor, while a bank with a net borrowing position would prefer a lower one, and the cases provide illustrative quotes of both incentives. Derivatives traders at numerous panel banks were found to have requested submissions aimed at benefiting their trading positions.[39] A Bank of Scotland trader explained:

> "Many institutions set their Libors based on their derivative reset positions."[40]

In a conversation on 6 July 2006, a Frankfurt Eurodesk manager of Deutsche Bank checks whether the manipulated rate would also benefit his colleagues in London:

> "Hihi [London MMD Manager], I just want to check whether we have conflicting interests in the June06 settlement. It doesn't make sense if we try to push one way and u wld like to have it the other way around. We wld prefer a low 3me fixing to push June06 high. Is this ur preference as well?"[41]

[37] U.S. Department of Justice, "Statement of Facts, Deferred Prosecution Agreement: Rabobank," 29 October 2013, page 33.

[38] European Commission, *Case A.39914—Euro Interest Rate Derivatives*, 4 December 2013, recital 32.

[39] See for example Financial Services Authority, "Final Notice: Barclays Bank PLC," 27 June 2012, pages 2-3.

[40] Quoted in Vaughan and Finch (2017), page 163 from an email of a Bank of Schotland trader to British Banking Association's Libor Director John Ewan that was shown to the jury in the Tom Hayes' trial.

[41] Commodity Futures Trading Commission, "Examples of Misconduct from Written Communication," 23 April 2015, page 2.

Contributing banks' money market desks would have been in a position to know the bank's overall net expose to the various rates and how they would gain or loose from movements in them. Internal documents from Deutsche Bank, for example, show that on 30 September 2008 Deutsche Bank tallied that it could gain up to €68 million for each basis point change in Euribor and Libor.[42] At UBS and later Citigroup, derivatives trader Tom Hayes, who was sentenced in the US to fourteen years in prison for dishonestly driving manipulation of Libor, stated the bank had software in place that calculated the exact effects of a change in Libor in each currency and maturity on trading profits.[43] At his trial, Hayes asserted that his managers had condoned his actions and UBS instructed company-wide that submissions be based on the bank's derivatives position spreadsheets.[44]

Hayes' hearing also gives insight into the volatility of trading exposures:

> "At various times you referred, when talking to others, to the difference that a movement of 1 basis point would make in respect of the substantial nominal sums you traded. Those figures varied from $\$500,000$ a basis point to $\$750,000$, $\$1$m a basis point to as much as $\$2.5$m a basis point."[45]

The exposure position a trader of bank faces on any given day is uncertain and largely stochastic, as it is the sum total of a vast number of transactions done by the banks' various trading desks worldwide. Around a smaller kernel of longer-term contracted money in- and outflows, exposure positions are largely driven by positions in OTC derivatives that are highly volatile.

2.1.4 Monitoring

The protocols by which the benchmark rates were produced by the British Banking Association (BBA) and the European Banking Federation (EBF) were vulnerable to collusion. The nature of the benchmark rates is that the fixings become public instantaneously, so that adherence to the agreement can easily be monitored. In a complete cartel, observing the averaged benchmarks suffices to detect defection by at least one cartel member. In addition, by publishing the individual submissions of each panel member identified together with the new rates, all cartel members could see what each of the others had submitted. Perfect monitoring is known to support the design of punishment strategies to stabilize collusion again individual cartel member's

[42] "Bank Made Huge Bed, and Profit, on Libor," *Wall Street Journal*, 10 January 2013.

[43] Vaughan and Finch (2017), page 23.

[44] At UBS an internal document titled 'Publishing Libor Rates' was recovered from the communal drive which contained such instructions. Vaughan and Finch (2017), page 154. Rabobank also admitted to participation of mid-level managers, including the Global Head of Liquidity and Finance, in the manipulation of Libor submissions. See U.S. Department of Justice, "Statement of Facts, Deferred Prosecution Agreement: Rabobank," 29 October 2013, pages 37-39.

[45] *R. v. Tom Hayes*, Southwark Crown Court, Sentencing Remarks of Mr Justice Cooke, 3 August 2015, recital 7.

temptation to unilaterally defect from the cartel agreement. Constant auditing of the individual bank submissions is standard practice for panel banks.

The Libor and Euribor rates were discussed upon publication.[46] Messages around their manipulations reveal instantaneous observation, as in this example from Deutsche Bank:

> "On 6 September 2006, Manager B contacted External Trader A and requested a low one month EURIBOR submission [...]
>
> On 7 September 2006, after the day's EURIBOR rates were published, the following exchange took place between Manager B and External Trader A:
>
> Manager B: "3.08!! thaaaaaaaaaaaaaaanks" External Trader A: "u see u see""[47]

Even though the fixing of the forex does not feature the same level of centralized information, the outcome of the forex rate could give it away if not the entire cartel respected the agreement. The effects of their manipulations, the conspirators monitored live in chatrooms, as this conversation between HSBC and RBS Traders transcripted by the CFTC makes clear:

> "15:51:57 Bank P Trader: good luck
>
> 15:52:20 Bank T Trader: load up your 50 offrs...
>
> 15:53:14 Bank P Trader: ill do those ones if you want
>
> 15:53:19 Bank V Trader: haah
>
> 15:53:20 Bank T Trader: ur fkg [Bank V Trader], ramp it
>
> [15:59:30 Fixing window opens]
>
> [16:00:30 Fixing window closes]
>
> 16:00:41 Bank T Trader: nice one *****
>
> 16:00:56 Bank P Trader: look at you!...-well done mate..."[48]

To the extent that WMR exercised it discretion to exclude some trades as non-representative, and if not all traders on the platform were involved in the cartel, monitoring from the fix values would not have been perfect.

[46] European Commission, *Case A.39914—Euro Interest Rate Derivatives*, 4 December 2013, recital 33.

[47] Financial Conduct Authority, "Final Notice: Deutsche Bank AG," 23 April 2015, page 17.

[48] Commodity Futures Trading Commission, "Examples of Misconduct in Private Chat Rooms", 12 November 2014, page 5.

2.1.5 Front running

After determining their actions collusively, and in the time window between when banks submit their Libor and Euribor quotes to the central administrator and when the new rates are being published, panel members could exchange information on their committed upcoming quote submissions before the new rate became public. This would allow the banks involved to adopt more favorable exposure positions through insider trading, at the expense of uninformed other market participants. It also would make alignment of panel members interests in colluding possible. There is no explicit discussion of front running in what has so far become public of the Libor and Euribor investigations. However, there is evidence suggesting that traders adjust their exposure to the rate depending on information on future quoting behavior. For example Hayes was quoted to have asked another submitter by email, referring to the bank's decision to increase its Libor submissions:

> "Do you talk to the cash desk and did we know in advance? We need good dialogue with the cash desk. They can be invaluable to us. If we know ahead of time we can position and scalp the market."[49]

Front running certainly was central in the forex manipulations.[50] Traders exchanged confidential information on their own positions and clients' trade orders, agreed beforehand in which direction they were going to push the rate in the upcoming fixing-window and planned how to transact to benefit from banging-the-close. Suppose that a trader would get client orders large enough to move the market at 3.30 p.m. to sell dollars for euros at the 4 p.m. fix. Executing it would lower the value of the dollar. The trader with coconspirators would have incentive to sell their dollar positions—or even going short in dollars—before the start of the 4 p.m. window. In the fixing-window, the cartel would then sell as many dollar client orders as possible, lowering the dollar benchmark at which the member later buy back dollars to their recover their positions. Without this banging-the-close, the dollar benchmark would have been higher and the client would have received more euros for his dollars. The traders benefitted from first selling at higher price and then buying back at a lower price. The opposite strategy allowed for collusively benefitting from large or combined buy orders to push up the 4 p.m. fix, at which later to sell the own currency positions.[51]

In addition, traders could use their knowledge about impending client orders to build up positions ahead of the fix to align interests:

[49] Vaughan and Finch (2017), page 114.

[50] See Commodity Futures Trading Commission, "Examples of Misconduct in Private Chat Rooms," 12 November 2014; "What is 'Front Running' and Why is there a Crackdown on Currency Traders?," *Financial Times*, 21 July 2016.

[51] In L. Vaughan, G. Finch and A. Choudhury, "Traders Said to Rig Currency Rates to Profit off Clients," *Bloomberg*, 12 June 2013, a trader interviewed estimates that a move in the benchmark of 2 basis points would be worth over $200.000 on a $1.3 billion client order.

> "Any members of the group with an opposing interest knew to offload their 'ammo' ahead of the fix to an unsuspected victim. Traders who weren't in the club were steamrollered mercilessly."[52]

Such a strategy constitutes a risk, as the price for dollars could increase after having created a short position, for example due to market-moving news or trading from other parties on the platform. By colluding with (enough) other banks, traders would have a larger effect on, and a better idea of the trading taking place in the fixing-window. Moreover, they would be able to profit from front running on any other cartel member's large client orders.

2.1.6 Eligible Transactions Rigging

A challenge to manipulation in forex is that banging-the-close is done with actual trades, and is therefore potentially costly. Before the reforms, the Libors and Euribors were predictions for which there was no prescribed method of estimation, or even a basis in actual transactions or historical values, so that it was easy and gratis to change the quotes. After full implementation of the reform proposals, they will be based on certain eligible actual transactions, but it is too early to tell patterns of manipulation.

However, the transactions that qualify as eligible have a small volume compared to the total exposure positions of panel banks. In particular, OTC derivatives, which are not eligible transactions for the calculation of the rate, are by far the largest asset class directly related to these interbank benchmark rates. The Financial Stability Board (FSB) reported in 2014 that over 170 trillion dollars in OTC derivatives are tied to the USD Libor, and over 197 trillion dollars to the Euribor. In comparison, the second highest and most USD Libor-related asset class are syndicated loans with an estimated 3.4 trillion dollars.[53] As banks' Libor or Euribor-related portfolio is so large compared to the volume of eligible transactions, banks could manipulate the latter to justify their rate submissions at relatively low costs for very large gain. Panel members could potentially even temporarily have swapped eligible transactions between themselves in order to built a submission basis, without sustaining the costs of suboptimal trades with outsiders.[54]

[52] Vaughan and Finch (2017), page 174.

[53] Financial Stability Board, "Market Participants Group on Reference Rate Reform, Final report," March 2014, pages 243 and 348.

[54] There is evidence of such trades that served no commercial purpose in Vaughan and Finch (2017). For example in a phone call to broker Farr at RP Martin, Hayes proposed: "If you've got any mates, mate, who will do you like a net trade, I can like basically give you like, fucking I don't know a trillion three-month Libor/Tibor and take back a trillion three-month Libor/Tibor. Obviously you'll net it with the other guy." (*op. cit.*, page 63). These practices are explained as so-called 'wash trades', with the purpose of paying commissions to brokers who helped influencing submissions of other banks. It would also be a means of carrying out matching eligible transactions at the intended cartel rate, avoiding costs of collusion.

Apart from the direct transactions costs involved in these trades, there would be indirect costs as well. The transactions that are now to underlie the Libor and Euribor submissions by construction trade against different rates than the current ones: the intended future ones, after all. In case of forex, pushing transactions through the fixing-window carries currency risks. An anonymous trader involved explained:

> "It could still backfire if another dealer with a larger position bets in the other direction or if market-moving news breaks during the 60-second window."[55]

Dealers therefore would only employ the strategy if they knew enough about the other banks' positions and had sufficiently large client orders:

> "Typically, that would need to exceed 200 million euros to have a chance of moving the rate, two of the traders estimated."

For these reasons it was attractive for traders to collude on front running client orders and concentrating trade when the benchmark rates are set—note that in and of itself, concentrating high volumes of trades around the fixing-window was not illegal. Likewise would lone attempts to support a different interbank rate be costly without much guarantee of being effective.

2.2 Reforms

In response to the Libor scandals, the FSA proposed reforms to the rate setting process that have since been implemented.[56] Since 2013, individual Libor quotes are no longer published simultaneously with the final rate. Maturities and currencies that were too thinly traded were discontinued, and a 3-month embargo was put on the publication of the individual submissions—Euribor quotes are still published simultaneously with the rates. Furthermore, the administration of Libor was transferred to ICE Benchmark Administration (IBA) in order to improve governance and oversight over the rate setting processes—and Euribor-EBF changed its name to the European Money Markets Institute (EMMI). The FSB subsequently called for underpinning the benchmarks with actual transaction data, which the IBA and EMMI are implementing.[57]

Libor contributions are now to be supported by actual trades at the submitted rates in unsecured deposits, commercial paper and certificate of deposit, where the

[55] L. Vaughan, G. Finch and A. Choudhury, "Traders Said to Rig Currency Rates to Profit off Clients," *Bloomberg*, 12 June 2013.

[56] HM Treasury, "The Wheatley Review of Libor: Final Report," 2012.

[57] See Financial Stability Board, "Market Participants Group on Reference Rate Reform, Final report," March 2014. Duffie and Stein (2015) also call for calculation on the basis of a wide set of transactions.

submitting bank received funding from wholesale market counterparties such as other banks, central banks and large corporations.[58] In principle only actual transactions done in the 24 hours prior to the submission deadline are eligible, but in case of insufficient transactions, a somewhat wider range can be used. Of all eligible transactions, the volume weighted average rate constitutes the rate submission, unless it is deemed unrepresentative by IBA, in which case it can adjust it.[59] The Libor remains to subsequently be calculated as the average of 50% of the submitted rates after trimming.

The Euribor will also evolve into a transaction based rate in the course of 2017.[60] Only transactions of unsecured cash deposits from specified counterparties and short-term securities, such as commercial paper and certificate of deposits, traded in the wholesale unsecured money markets in the past 24 hours are eligible in the volume-weighted average rate submissions. In case of insufficient transactions, eligible transactions from previous days can be included as well. As a last resort, EMMI can take responsibility for setting the benchmarks. Contrary to Libor, banks have no residual, non transaction-based, discretion over their submissions. Euribor would subsequently be calculated as the average of the middle 4 or 5 quotes, or with the current 20 banks to which the Euribor panel has gone down since the scandals, 75% trimming. Yet EMMI intends to grow the Euribor panel again to approximately 40 members.[61]

While these reforms make the Libor and Euribor more robust, the forex manipulations have shown that even an entirely transaction-based rate could be colluded to increase trading profits. The eligible transactions are only a small part of the bank's portfolio exposures to the benchmarks.[62] Banks could transact those at rates that are suboptimal on their own, but support manipulations of the benchmarks that are beneficial to the larger rest of their trading books.[63] Manipulation with transactions remains possible with the reformed forex as well, to which the main adaptation has only been to base the fixes on transactions collected from a somewhat longer time window: between 2.5 minutes before and after 4 p.m., instead of a 1 minute window

[58] ICE Benchmark Administration, "IBA Libor Position Paper," 20 October 2014; ICE Benchmark Administration, "Second Position Paper on the Evolution of ICE Libor," 31 July 2016.

[59] IBA also studies the feasibility of a centrally calculated rate, where banks would only submit raw data on eligible transactions. See ICE Benchmark Administration, "Roadmap for ICE Libor," 18 March 2016, pages 20-21.

[60] See European Money Markets Institute, "Euribor Code of Conduct," 1 June 2016; European Money Markets Institute, "The Path Forward to Transaction-based Euribor," 21 June 2016, pages 11-12.

[61] European Money Markets Institute, "The Path Forward to Transaction-based Euribor," 21 June 2016, page 13.

[62] By far the largest volume of transactions tied to Libor and Euribor is from over-the-counter derivatives, which are not eligible for the calculation of the rate. See Financial Stability Board, "Reforming Major Interest Rate Benchmarks," July 2014, page 348.

[63] The possibility that eligible transactions would be rigged for the purpose of unilateral manipulation was suggested in Duffie and Dworczak (2014).

around the hour.[64] Monitoring of individual submissions remains perfect in Libor and Euribor, even though the delay in publishing potentially makes deviations more difficult to detect. Despite implemented and proposed reforms, the benchmark rate setting processes remain vulnerable to collusion.

3 A Model of Benchmark Collusion

In this section we develop a model to show how collusive interbank lending benchmark rates setting based on actual transactions, i.e. as it is after implementation of the reforms, could be possible. Slightly amended, the model also applies to collusively banging-the-close in forex, as those cartel mechanisms similarly rely on exchanging inside information, aligning exposure positions and planning eligible transactions.[65]

Consider a panel of N banks $i = 1, ..., N$ that interact on each trading day for an infinite period. Let v_{0it} be bank i's baseline portfolio position by which it is exposed to changes in the interbank rate for a certain maturity in period t, and c_{0it} its true eligible transaction rate, which represents the bank's true borrowing costs. Position and rate are bank-specific, volatile over time and largely stochastic. Its baseline portfolio position is the overall net trading book exposure of the bank, reflecting all the directly Libor-related activities it is involved in. Variations in the eligible transaction rates reflect market changes over time, as well as that banks generally differ in various dimensions that may affect the rates at which they are able to borrow on the money market, such as capital structure, solvency and liquidity.

At the beginning of the trading day, the current interbank lending rate is L_{t-1} is known and that for the coming period L_t is to be determined on the basis of all panel banks' rate submissions c_{1it}, $i = 1, ...N$. During a trading day, each bank can adjusts both its exposure to changes in the interbank rate and its eligible transaction rate. Let these adjustments be Δv_{it} and Δc_{it}, respectively, so that v_{1it} is the realized exposure of bank i at the end of period t and c_{1it} its realized eligible transaction rates, with $v_{1it} = v_{0it} + \Delta v_{it}$ and $c_{1it} = c_{0it} + \Delta c_{it}$.[66] We denote the vector of all banks' values

[64] See Thomson Reuter, *WM/Reuters FX Benchmarks: Sport & Forward Rates Methodology Guide*, January 2017. The Foreign Exchange Working Group at the Bank for International Settlements (BIS) is also establishing a single global code of conduct, which is foresaid to be published in the course of 2017. See Bank for International Settlements, "FX Global Code: May 2016 Update."

[65] In the case of forex, an individual trader's influence on the rate is somewhat more complex. The number of traders is larger and can vary, and a larger bank is likely to have a higher number of trades and therefore a larger effect on the rate than a smaller bank. Also, since the transaction prices are not weighted by volume, traders can increase their impact on the rate by breaking up larger orders into a number of smaller ones to execute separately. At the same time can a smaller number of large banks move the forex rates more than the Libors or Euribors, where each bank's contribution weights equally. Moreover, banging the close is a form of eligible transactions rigging that is profitable in and of itself. Details of how the main features of the model apply to forex collusion are given in footnotes.

[66] For forex, c_{0it} are the true incoming client orders and Δc_{it} are withheld, accumulated or rejected

in period t by leaving out the subscript i, e.g. $(v_{0t}, c_{0t}) = (v_{01t}, ..., v_{0Nt}, c_{01t}, ..., c_{0Nt})$ and $(\Delta v_t, \Delta c_t) = (v_{1t}, ..., v_{Nt}, c_{1t}, ..., c_{Nt})$.

Assuming that each panel bank always submits a quote each period, the new interbank rate L_t is determined as the trimmed average of all N quotes.[67] We call the set of submissions from which the upper and lower share of ranked quotes are discarded the 'trimmed range' $\mathcal{T}$ consisting of n banks. Hence,

$$L_t = \frac{1}{n} \sum_{j \in \mathcal{T}} c_{1jt}, \tag{1}$$

where c_{1it} is based on bank i's final set of eligible transactions. Note that for Euribor and before the Libor embargo was introduced, c_{1it} used to be published for all banks i simultaneously with L_t—since the Libor embargo, this is with a delay.[68]

Since the majority of financial contracts, such as swaps, futures and corporate loans, have linear payouts, bank i gains from a change in the rate from the current to the next trading day by

$$\pi_{it} = v_{1it} \cdot (L_t - L_{t-1}) - C(\Delta v_{it}, \Delta c_{it}), \tag{2}$$

where $C(\Delta v_{it}, \Delta c_{it})$ are any costs associated with bank-specific changes in exposure and rate. Note that its eligible transactions, which in practice are small in comparison, are not part of a bank's exposure to the benchmark rate.[69]

Choice variables $(\Delta v_{it}, \Delta c_{it})$ are based on bank i's expectations about the development of the rate. If bank i would have inside information about the value of new rate before it is published and adjust its portfolio position on the basis of that information, choice variable Δv_{it} reflects front running. Even though parts of banks' exposure profile, such as that relating to mortgages and other long-term contracts, is relatively stable over time, trade in OTC derivatives is vast large and fast-changing in comparison. The bank engages in eligible transactions rigging if it does eligible transactions not at the true borrowing costs c_{0it}, but at rates that are intended to support a particular submission c_{1it}.

client orders, as well as possibly fake transactions with other cartel traders. Gains on the own exposure v_{0it} by collusion result from selling or buying before the fix, with matching reverse orders after. Both position and rate are bank-specific and (at least in part) exogenous.

[67] Formally, the Libor and Euribor calculation methods allow for a minimum number of submissions to calculate the rate (in the case of Libor: 5 quotes, for all of the tenors for a particular currency; in case of Euribor: 50% or 12 banks, from at least 3 different countries), so that in practice a cartel could save on manipulation costs by agreeing the that banks with the highest cost to do submit a quote at all. In practice, the minimum rule is meant for exceptional circumstance and banks systemtically not submitting would raise suspicion with the authorities, even prior to the scandals.

[68] The forex rate would be the median rate of all trades collected and included as valid by WMR during the fixing-window. As WMR has discretion over which rates to include, there is some uncertainty for a trader about his exact influence on the rate.

[69] In the case of forex, if a bank were to engage in transactions during the fixing-window on its own account, this would affect its exposure position to the rate.

Front running is costly because of direct transaction costs, trade risks in predicting the final rate and liquidity constraints. Since the Libor and Euribor reforms, eligible transactions are required to support a submission and rigging them requires a bank to trade on otherwise suboptimal terms.[70] Extreme manipulation may further be constrained by the risk of raising suspicion of manipulation with other market parties and regulators. Before the reforms, when the quotes were estimations with no actual transactions, getting caught would have been the main source of manipulation costs.[71] It is natural to assume that $C'_{\Delta v_{it}}(\cdot)$ and $C'_{\Delta c_{it}}(\cdot)$ are both positive and increasing. In addition, we assume that the functional form of $C(\Delta v_{it}, \Delta c_{it})$ is such that a global maximum for π_{it} exists and is unique.[72]

The banks play an infinitely repeated daily simultaneous move game. Each bank is privately informed of its baseline exposure v_{0it} and eligible transaction rate c_{0it}. If the banks behave non-cooperatively and do not share their baseline values, one possibility is that they behave according to the code of conduct and submit their true eligible transaction rate c_{0it}. Alternatively, each bank may maximize its own expected payoff given its prior on the baseline values of the other banks, playing static Bayesian-Nash.

Through L_t, the payoff function of each bank depends not only on its own exposure and eligible transactions, but also on the eligible transactions of the other banks in $\mathcal{T}$, which in turn depend on their exposures. As a result, there is an incentive to collude and coordinate behavior. If the panel colludes, these values (v_{0t}, c_{0t}) are shared and the cartel determines the joint-profit-maximizing front running and eligible transactions rigging strategies collectively.

Figure 2 illustrates the timing of cartel events in the Libor rate-setting process. OB and CB are the opening and closing bells of the trading day at the London Stock Exchange, at 8.00 a.m. and 16.30 p.m. GMT, respectively. At opening, L_{t-1} is the current Libor rate. Suppose that at time 0_t, shortly into day t, banks learn what will be their own true borrowing costs c_{i0t} and baseline exposure position to the new rate v_{i0t}.[73]

[70] In the case of forex, manipulations with actual transactions typically are not suboptimal, but rather the source of cartel profits. The costs involved for Δc_{it} would be missed commissions in case of withholding and discovery, and for Δv_{it} for transacting and currency risks.

[71] Vaugh and Finch (2017) state that Hayes maintained that he only made requests for Libor movements within a boundary of where cash would realistically be trading—the so-called 'permissible range': "If he'd asked a trader at another firm to ramp his submission by several basis points, Hayes said, his reputation in the market would have been undermined." Documents of UBS however showed that Hayes' managers had ordered much more significant distortions of the rate. *Op. cit.*, page 153.

[72] Since the first part of π_i is linear in both Δv_{it} and Δc_{it}, together with positive and increasing marginal costs, a necessary and sufficient condition for global maximum is that the product of the two second-order partial derivates of $C(\Delta v_{it}, \Delta c_{it})$ is larger than the square of the cross-partial derivate, that is $C''_{\Delta v_{it}\Delta v_{it}}(\cdot) \times C''_{\Delta c_{it}\Delta c_{it}}(\cdot) - C''_{\Delta v_{it}\Delta c_{it}}(\cdot)^2 > 0$ under the assumption that $C''_{\Delta v_{it}\Delta c_{it}}(\cdot) = 0$, which is natural since the two manipulation mechanisms relate to very different transactions.

[73] Note that although illustrated in Figure 2 at a specific point in time (0_t shortly after OB), in

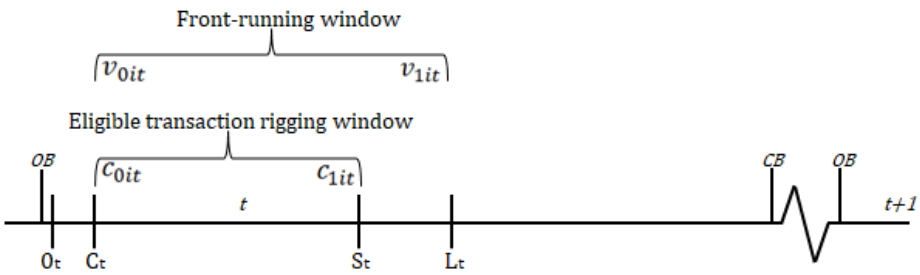

Figure 2: A trading day in the life of Libor.

Subsequently, the banks consider alternative front running (Δv_{it}) and eligible transactions rigging strategies (Δc_{it}): either none, non-cooperatively or cooperatively. If they collude, the panel banks share their private information at cartel meeting C_t in which each member's designated joint-profit-maximizing front running and eligible transactions rigging is determined. Latest at 11.00 a.m., moment S_t, all banks submit their Libor quote based on c_{i1t}, which closes the window for eligible transactions rigging.[74] The window for front running remains open longer, until publication at 11.55 a.m. of the new rate at L_t.[75]

We assume that both v_{0it} and c_{0it} are daily i.i.d. private values drawn for each bank i from symmetric and commonly known continuous distributions. This setup captures market volatility, while limiting the analysis to myopic profit maximization. The overall exposure position of the panel banks to the rate is assumed to fluctuate around zero, i.e. $E[v_{0i}] = 0$, capturing that it mostly stems from transactions in OTC derivative markets, which are volatile and liquid enough for all banks to regularly find themselves flipped from one side of the market to the other and back again. The assumption is also consistent with the Libor time-series displaying a fixed mean pattern. If the panel banks instead had an exposure profile with a persistent mean in one direction, they would have incentives to drift the Libor rate in that direction,

practice the banks see their baseline values change continuously, as trading takes place around the clock worldwide and unforeseen events or market-moving news could constantly affect the baseline values. The cartel can accommodate such multiple changes by sharing the relevant information and updating the cartel strategy throughout, as long as the windows for manipulation are open.

[74] Note that eligible transactions for Libor submissions on day t are those executed between the previous submission and the new submission at S_t. Collusive eligible transactions rigging can only be done after information has been exchanged at time C_t. For Euribor submissions on day t, all transactions executed on trading day $t-1$ are eligible. Therefore, the eligible transactions rigging window for Euribor is somewhat different from the one in Figure 2. The Euribor cartel would use earlier baseline information and need to meet earlier, so as to manipulate eligible transactions the day before.

[75] For Libor, the difference between the eligible transactions and front running windows is less than an hour: quotes are submitted by the end of the eligible transactions window at 11.00 a.m. and the rate is published at 11.55 a.m.. In the case of Euribor, the difference between the eligible transactions rigging- and front running windows is the time between the end of trading day $t-1$ and when the rate is published shortly after 11.00 p.m. on day t.

which is not observed

The baseline eligible transaction rates c_{0it} are assumed to have mean $E[c_{0i}] = L_{t-1}$, that is the previous day interbank rate. Instead of taking the realized interbank rate of the previous day (based on c_{1it-1}), it is also possible to take the unmanipulated interbank rate instead (based on c_{0it-1})—although it is not obvious how this information would be available for non-cartel banks once the Libor is doctored. Alternatively, a constant mean could also be assumed. Also for the special case of the panel banks being perfectly homogenous, all results hold straightforwardly for $c_{0it} = L_{t-1} + \varepsilon_t$ with certainty, where ε_t is some common shock.

3.1 Independent Quoting

If the panel banks formulate their contributions independently, they do not share private information and their portfolio changes and submissions are determined playing an infinitely repeated simultaneous move stage-game of incomplete information. Let bank i's strategy if all banks follow the banking code of conduct and accordingly honestly submit their true borrowing cost and not front run, be $\Delta v_{it}^* = \Delta c_{it}^* = 0$ with π_{it}^*. Then $E_{it}[\pi_{it}^*] = E[\pi^*] = 0$ for all $i = 1, ...N$ and $t = 1, ..., \infty$, which follows directly from symmetry and the i.i.d. assumptions—the $*$ indicating honest quoting.

To follow the banking code of conduct is not individually optimal, however. Instead, maximizing own expected profits within the benchmark system, i.e.

$$\max_{\Delta v_{it}, \Delta c_{it}} E_{it}[\pi_{it}], \tag{3}$$

potentially induces each bank to engage in front running or eligible transactions rigging. Manipulation is unilateral, assuming that the other panel members report honestly, since banks have no information to expect each others actual exposures to be short or long. Equilibria are in pure strategies. Let π_{it}^{BN} be the payoff of bank i in period t in the static Bayesian-Nash equilibrium, with expected value $E_{it}\left[\pi_{it}^{BN}\right]$. From symmetry and the i.i.d. assumptions is follows that $E_{it}\left[\pi_{it}^{BN}\right] = E\left[\pi^{BN}\right] \geq 0$ for all banks i the same, each period.

3.2 Collusive Quoting

If the panel banks would form a cartel instead, they share the information (v_{0t}, c_{0t}) each day prior to the rate setting process truthfully. With this inside information, the cartel then determines cooperatively what joint-profit-maximizing new rates to establish, by how much each member is to engage in eligible transactions against the target new rate to support the collusive submissions, and how each member is to front run. The changes to the banks' portfolio positions are designed to give the members more beneficial exposure to the future rate and reduce misalignment of exposures between them.

The cartel strategy in period t follows from maximizing joint profits under complete information

$$\max_{\Delta v_t, \Delta c_t} \sum_{i=1}^{N} \pi_{it}\big|_{(v_{0t}, c_{0t})}, \tag{4}$$

where Δv_t and Δc_t are vectors of advised positions and submission targets.

Optimal eligible transactions rigging is executed before the new quotes are submitted. Front running remains possible also after, until the new rates become known to all market participants. We obtain that the full panel can agree on a collusive strategy.

Proposition 1 *There exists an optimal cartel strategy* $\left(\Delta v_t^C, \Delta c_t^C\right)$*, in which* $c_{0(i+1)t} + \Delta c_{(i+1)t} \leq c_{0it} + \Delta c_{it} \quad \forall i = 1, ..., N-1$.

Proof. As a necessary condition for joint profit maximization, the cartel will minimize collusion costs $\sum_{i=1}^{N} C\left(\Delta v_{it}, \Delta c_{it}\right)$. As part of the equilibrium conditions, the marginal bank-specific costs of changes in the eligible transaction rate are assumed to increase in Δc_{it}, i.e. $C''_{\Delta c_{it}} > 0$, and $C''_{\Delta v_{it} \Delta c_{it}}(\cdot) = 0$. Therefore, if the cartel would change the ranking of the eligible transaction rates, the same set of final rates c_{1t} could have been achieved at lower total eligible transaction rate rigging costs by retaining the ranking. This implies that collusion does not change the ranking from the baseline values, or for the joint-profit maximizing outcome it holds that:

$$c_{0(i+1)t} + \Delta c_{(i+1)t} \leq c_{0it} + \Delta c_{it} \quad \forall i = 1, ..., N-1, \tag{5}$$

where bank indicator i is equal to its rank based on the baseline eligible transaction rates c_0. Since the baseline eligible transaction rates c_0 are drawn from a continuous distribution F_c, with probability 1 $c_{0it} \neq c_{0jt}$ for all $i \neq j$, a unique ranking of baseline rates is possible. Therefore without manipulation all constraints hold, so that $(\Delta v_t, \Delta c_t) = (\emptyset, \emptyset)$ is a Slater point, the existence of which is both necessary and sufficient for the existence of a global optimum, $\left(\Delta v_t^C, \Delta c_t^C\right)$, in this non-linear optimization problem with inequality constraints—as shown, for example, in Brinkhuis and Tikhomirov (2005), pages $210 - 211$. ■

Which banks are included in $\mathcal{T}$ and which are not depends on the portfolio drawings and so varies over time. Yet the cartel will designate the same subset of the panel banks to move the rate that would be in $\mathcal{T}$ if the submissions would be formulated honestly.[76]

Let $\pi_{it}^C = \pi_{it}\left(\Delta v_t^C, \Delta c_t^C\right)$ be bank i's payoff in the cartel optimum in period t, and $E_{it}\left[\pi_{it}^C\right]$ bank i's period t expected profits from participating in the cartel before

[76] Note that in case of Bayesian-Nash independent quoting, it may happen that different banks turn out to be in the trimmed range $\mathcal{T}$, as in that case the banks manipulate without knowing each others positions and behaviours.

information sharing. The latter are equal for all banks and all periods, since all baseline values (v_{0it}, c_{0it}) are assumed to be drawn from the same distributions, with the mean of the distribution of baseline eligible transaction rates being L_{t-1} and the variance constant. Exposure positions and true borrowing costs are reset every day, so that all banks have an equal chance both of being on the right side of the market, and of having to incur the costs of eligible transactions rigging. Note that, since it is always possible for a bank participating in the cartel at least to front-run, $E_{it}\left[\pi_{it}^{C}\right] = E\left[\pi^{C}\right] > E\left[\pi^{BN}\right] \geq E\left[\pi^{*}\right] = 0$.

As expected profits are equal within the cartel, no explicit side-payments are necessary: while one day certain panel members bear the larger part of the cost of colluding and others gain most, the next this may be the other way around, and over time all banks expect to profit equally. Cartel members in turn 'take one for the team' by submitting quotes that are not optimal given their baseline exposure position. Yet the information on the positions and submissions of the others enables them to mitigate the losses by front running. In fact, the ability to create movement in the rates that is predictable with inside information after the exchange of information alone makes a cartel beneficial. As counterparties trading in financial products tied to the rate are less well-informed of where the future rate will go, cartel members can profit at their expense.

Banks both inside and outside $\mathcal{T}$ will always find it in their interest to front-run, as part of the cartel strategy. In addition, banks outside $\mathcal{T}$ will occasionally be called upon to engage in eligible transactions rigging and incur the costs of collusion if the joint cartel optimum requires them to manipulate their submission. This can happen if, in case banks outside $\mathcal{T}$ maintain their quote whereas the collusive submission strategy requires more extreme quotes, they would effectively become part of $\mathcal{T}$ while pushing the designated more extreme quoting cartel members out of the trimmed range. These banks manipulate their rates in order for them to move over, be trimmed off, and so accommodate the rigging by banks in $\mathcal{T}$.

Figure 3 illustrates such a situation in the case of four panel banks, the middle two of which are in the trimmed range (so $N = 4$ and $n = 2$).

Bank 1 moves over to the right, so that banks 2 and 3 together can drive up L_1 as their average submission. Note that Proposition 1 implies that it is never (unconstrained) optimal for banks in the periphery (banks 1 or 4), to cross over into $\mathcal{T}$ and move the rate instead of the banks with an interior position, as this is always more costly.

Note that assuming the panel members report their true borrowing costs and baseline exposure positions to the cartel truthfully is not that stringent, in the sense that it is not obvious how a bank would be better off lying. In principle, banks reporting different than actual values would not be easily discovered, as long as they subsequently behave according to cartel instructions. A motive could be to try to avoid cost of collusion, by pretending to have relatively low or high borrowing costs,

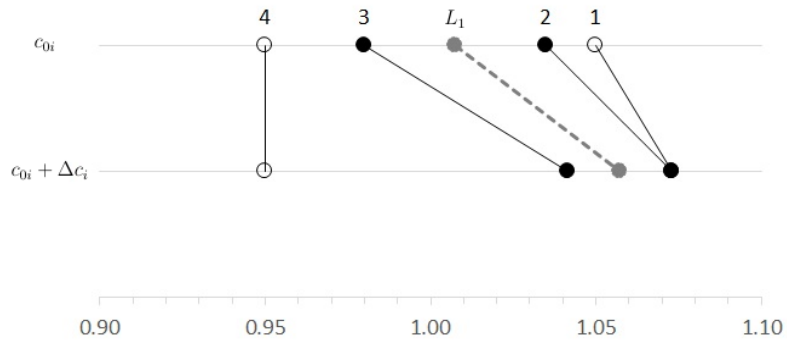

Figure 3: Panel bank 1, not included in $\mathcal{T}$, engaging in collusive eligible transactions rigging.

or increase chances of the optimal cartel quote being in its favor, by overstating its exposure position. However, as long as the cartel makes all banks report their private information on positions and rates simultaneously, so that none knows the other banks' exposures in advance, reporting other than true costs may just as well land a bank at the wrong side of true borrowing cost, so that it ends up being assigned higher eligible transactions rigging costs than it would with the truth. Similarly, by overstating its exposure position a bank risks too extreme optimal cartel quotes, for which its manipulation costs exceed the gains from trying to manipulate the cartel agreement.

3.3 Deviation

After banks have shared their portfolio positions and determined the optimal cartel strategy for the day, each bank can have an incentive to unilaterally deviate. For instance, one or more banks within the trimmed range $\mathcal{T}$ may have a negative exposure to $L_t - L_{t-1}$, but still be designated to facilitate upwards rigging of L_t for the benefit of the cartel. Also, a bank with an exposure position that is in line with the cartel but relatively small may incur more costs from the manipulation, which from the bank's perspective is too extreme, than benefits from its trading books. By unilaterally defecting, a bank in such a position would benefit from reducing the upward manipulation of L_t, while foregoing the costs of its eligible transactions rigging, taking position on the new rate based on his own unique inside-information about his defecting submission—at the expense of the other cartel member banks.

Bank i's optimal deviation in period t follows from maximizing its profits given that the other cartel members play the collusion strategy

$$\max_{\Delta v_{it}, \Delta c_{it}} \pi_{it}\big|_{\left(\Delta v^C_{-it}, \Delta c^C_{-it}\right)}, \tag{6}$$

in which Δv^C_{-it} and Δc^C_{-it} refer to the front running and eligible transactions rigging

of all panel members but bank i under the collusive optimum.

While a defecting bank can decide to position itself at any point within the trimmed range $\mathcal{T}$, so that its final eligible transaction rate is included in the derivation of the new interbank rate, this need not be optimal, depending on the bank's position. For example, if bank 4 in Figure 2 had a negative exposure, it would want to see the new rate as low as possible, whereas positioning itself within $\mathcal{T}$ would only result in a (weakly) higher rate and positive eligible transactions rigging costs. It follows that a deviating bank will either position itself in $\mathcal{T}$ in order to attempt to influence the rate, or it does not engage in transactions, for it knows its submission will be trimmed off—whichever gives higher payoff. That is,

$$c_{0it} + \Delta c_{it} \in \mathcal{T} \quad \text{or} \quad \Delta c_{it} = 0. \tag{7}$$

We denote bank i's optimal deviation payoff by $\pi_{it}^D = \pi_{it}\left(\Delta v_{it}^D, \Delta c_{it}^D, \Delta v_{-it}^C, \Delta c_{-it}^C\right)$. It can be large compared to π_{it}^C, in particular for banks with large positive or negative portfolio positions.

Once the Libor is published, all the cartel members can immediately infer from the rate whether there has been defection from the cartel agreement—as well as which bank(s) deviated from the individual submissions published, after the embargo. It is not obviously possible to observe for the cartel whether a bank has deviated from the agreed position changes, yet these are individually optimal to implement for a cartel member that follows the agreed submission strategy—and deviations in Δv_{it} have no effect on the profits of other cartel members anyway.[77]

3.4 Cartel Stability

The cartel would need to stabilize adherence to its agreements against incentives to deviate. That is, it plays the per-period strategy that maximizes joint profits, subject to the constraints that for each bank i the expected value of collusion (V_{it}^C) is at least as high as the expected value of deviation (V_{it}^D). Using π_{it}^C, π_{it}^D and π_{it}^{BN} and discount rate $\delta \in (0, 1)$, we can specify for bank i in period t

$$V_{it}^C = \pi_{it}^C + \delta E\left[V^C\right], \tag{8}$$

in which $E\left[V^C\right]$ is the expected continuation value of collusion, and

$$V_{it}^D = \pi_{it}^D + \delta E\left[V^P\right], \tag{9}$$

as the instantaneous profit from deviating plus the expected value of its consequences once discovered and punished, $E\left[V^P\right]$, which is immediately.

Suppose that the cartel has a punishment strategy in place in which defection triggers $T \geq 0$ periods of reversion to non-cooperative contributions, so that

[77] In the case of forex, monitoring adherence to the cartel is somewhat imperect, due to WMR's discretion over which rates to include in calculating the median.

$$E\left[V^P\right] = \sum_{t=0}^{T-1} \delta^t E\left[\pi^{BN}\right] + \delta^T E\left[V^C\right]. \tag{10}$$

To assure adherence to the cartel by bank i, in each period t the panel banks then maximize joint profits (4), subject to $V_{it}^D \leq V_{it}^C$, or

$$\pi_{it}^D - \pi_{it}^C \leq \left(1-\delta^T\right)\delta E\left[V^C\right] - \sum_{t=1}^{T} \delta^t E\left[\pi^{BN}\right] \quad \forall i = 1, ..., N, \tag{11}$$

in which the left-hand side payoff differentials of equation (11) vary between banks and periods, depending on the portfolio positions, and the right-hand side is a fixed critical cut-off value that decreases in δ and increases in T. Note that if these incentive compatibility constraints hold for Bayesian-Nash independent quoting, they certainly do also for honest quoting, since $E\left[\pi^{BN}\right] \geq E\left[\pi^*\right]$.

As defection never happens in equilibrium, and therefore punishment is not costly, the cartel optimally uses a grim trigger strategy with infinite punishment.

Proposition 2 *Punishment period $T \to \infty$.*

Proof. Since the cartel maximizes (4) subject to (11) in each period, deviation is not an equilibrium strategy and it is optimal to maximize punishment in order to ensure adherence to the cartel agreement. A stable cartel choice set that can be sustained with lower punishments will also be sustainable under more severe punishments. The right-hand side of each constraint (11) is increasing in T, meaning it is less restrictive when the punishment period is longer. Therefore it is optimal for the cartel to choose an infinite punishment length. ■

3.4.1 Continuous Collusion

The optimal cartel strategy would be continuous collusion, in which the cartel each period adjusts the profit maximizing vector of rate submissions, associated eligible transactions rigging and front running to the incentive compatibility constraints as they materialize from the individual bank portfolio positions and rates drawn. The cartel can maintain continuous collusion by responding optimally in each period to keep each payoff differential below the critical value, given δ and T, by adjusting π_{it}^C, and indirectly also π_{it}^D in the incentive compatibility constraints (11). It can do so by constraining $\left(\Delta v_t^C, \Delta c_t^C\right)$. If one or more banks happen to draw large value positions, positive or negative, the cartel tempers the optimal new rate accordingly, whereas without extreme drawings it can rig the rate more extremely. Note that the optimal cartel strategy from solving this Kuhn-Tucker problem with constraints that are best-responses to any possible collusive outcome includes possibly requiring inefficient production of the benchmark rates. That is, the cartel agreement may

require higher than minimal total costs of collusion, in order to keep stability by reducing the payoff differential for cartel members with a strong temptation to defect.

Observe also that the continuous collusion strategy of the benchmark rate cartel is more complex than that for cartels in conventional markets as in Rotemberg and Saloner (1986), in which the optimal defection profit is the common entire collusive profit for the period. Here, we generically have asymmetric payoff functions that provide N different inequality constraints, each of which itself results from the optimization problem by which each bank determines its optimal deviation strategy π_{it}^{D} for its portfolio position and rate, given that all other panel banks behave according to the cartel agreement.

Rotemberg and Saloner (1986) rely on the cartel's option to fall back during booms, when the incentive to deviate is largest, on the competitive outcome of pricing at marginal costs, from which no cartel member would deviate. In essence, by lowering the collusive price and thus profits, the payoff differential from deviation becomes lower and the cartel remains stable under infinite punishment. Such a fixed fallback option does not exist in our model of benchmark rates collusion, since the incentives to deviate vary with individual positions and rates, which the cartel members share in the beginning of each period as common knowledge. For example, should the cartel entertain to default to honest quoting, cartel members would have an incentive to deviate on the basis of their ability to predict the new rate and other panel members' optimal front running. While portfolio position-specific stable collusive actions may exist for any two vectors of drawings (v_{0t}, c_{0t}), these would need to be determined every period anew.

Doing so is further complicated by the trimming mechanism, which introduces discontinuities in the problem, as a large number of different cases can arise and need to be internalized in each of the bank's decisions. For example may banks that used to be in the trimmed range $\mathcal{T}$ be trimmed out after defecting, while other banks may become part of $\mathcal{T}$. Such switches cause jumps in the profitability of both the cartel and the defecting bank. From conditions (5) above, in up to 2^{N-1} cases, the cartel needs to check for each of these conditions whether it is binding or slack, which makes an analytical derivation of a solution involved, even for low values of N.[78] In addition, since the drawings of (v_0, c_0) are from continuous distributions on unbounded supports, the domain of Δv and Δc is not compact. These complexities imply that existence of a global cartel optimum is not guaranteed. Also from a practical computational point of view, the optimal cartel problem is highly complex to solve daily, so that the first-best optimization strategy of maximizing joint payoff subject to cartel stability may not be feasible.

[78] Brinkhuis and Tikhomirov (2005), page 211.

3.4.2 Episodic Break-up

The benchmark cartel can avoid the computational complexities in determining the continuous collusion strategy by using an episodic break-up strategy, as in Fershtman and Pakes (2000). All panel banks choose the unconstrained joint profit maximizing strategy as long as it satisfies all bank's incentive compatibility constraints per period, until at least one panel bank will deviate, in which case all banks revert to non-cooperative contributions for that period. The cartel is continuous in that each period information is shared, but also breaks up episodically for unstable periods. Only deviation from this strategy would be punished with reversion to non-collusive contributions for ever after. This is again off-equilibrium.

During a break-up, the panel banks have inside information to determine their contributions non-cooperatively. Let the one-period static Nash equilibrium with full information of the panel banks be π_{it}^{N}, with $E\left[\pi_{it}^{N}\right] = E\left[\pi^{N}\right]$ for all $i, ...N$, given the assumptions on portfolio distributions. Since interests are typically conflicting, the Nash equilibrium need not be unique, nor exist in pure strategies. Note that $E\left[\pi^{N}\right] \geq E\left[\pi^{BN}\right]$, since all banks are fully informed in formulating the break-up contributions and any information that helps a bank to better predict the new rate allows it to front-run lucratively and increase expected payoff. Without agreement on the rate, banks can only front run in the direction of where they expect the rate to go. Generally, their portfolio changes will be more conservative than under full collusion.

To analyze the pattern of switching between full collusion and episodic break-ups, let $\rho \in [0, 1]$ be the probability that the unconstrained joint-profit-maximum violates one or more of the incentive compatibility constraints and the cartel reverts to one-period static Nash. Per-period expected payoff from colluding then is

$$(1-\rho)\, E\left[\pi^{C}\right] + \rho E\left[\pi^{N}\right], \tag{12}$$

where $E\left[\pi^{C}\right]$ is conditional on there being no break-up, and $E\left[\pi^{N}\right]$ on there being a break-up. Since break-up occurs at extreme value positions that the panel bank(s) can exploit with inside information, it may well be that $E\left[\pi^{N}\right] > E\left[\pi^{C}\right]$

Given Proposition 2, the net present value of all forgone future expected payoffs in case of deviation comes

$$\frac{\delta}{1-\delta}\left((1-\rho)\, E\left[\pi^{C}\right] + \rho E\left[\pi^{N}\right] - E\left[\pi^{BN}\right]\right) = \Psi\left(\delta, \rho\right), \tag{13}$$

since only in a punishment phase is quoting truly non-collusive, resulting in π_{it}^{BN}—or possibly π_{it}^{*} if the panel banks choose to follow the code of conduct.

The probability ρ of episodic cartel break-up is now defined implicitly by the tightest stability constraint,

$$\rho = 1 - Pr\left[\max_{i=1,..,N}\left(\pi_{it}^{D} - \pi_{it}^{C}\right) \leq \Psi\left(\delta, \rho\right)\right]. \tag{14}$$

Given $\Psi(\delta, \rho)$, the value of ρ is under the remaining tail of the probability density function of $\max_i \left(\pi_{it}^D - \pi_{it}^C\right)$, which derives from the distributions over bank i's initial portfolio position v_{0i} and eligible transaction rate c_{0i}. Figure 4 illustrates.

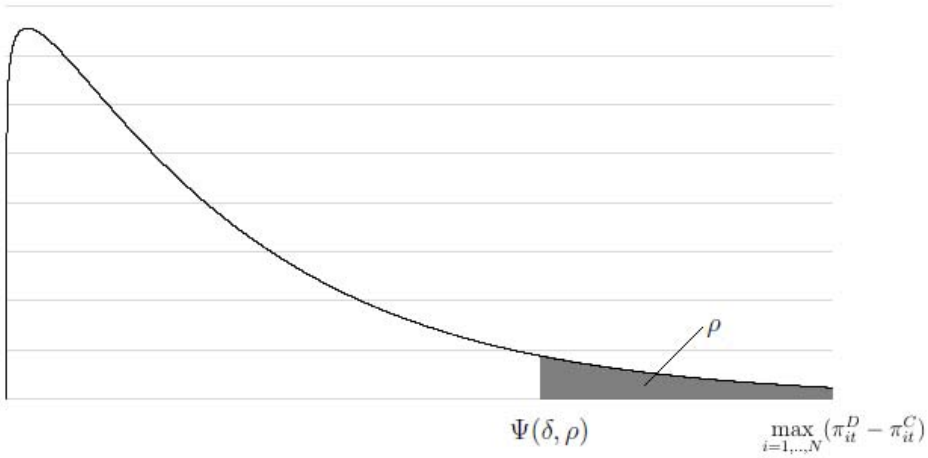

Figure 4: Cartel break-up probability defined by the payoff-differential distribution.

We can now establish conditions for the existence of stable continuous collusion with episodic break-up.

Proposition 3 *For a continuous and sufficiently widely supported distribution of the maximum payoff differential, ρ is strictly between* 0 *and* 1.

Proof. The implicit definition of ρ in equation (14) is a continuous mapping from a nonempty, compact and convex set $\rho \in [0, 1]$ onto itself, so that at least one fixed point solution exists. Let the support of the continuous distribution of $\max_i \left(\pi_{it}^D - \pi_{it}^C\right)$ be $[a, b]$. For a lower bound $a < \Psi(\delta, 1) = \frac{\delta}{1-\delta}\left(E\left[\pi^N\right] - E\left[\pi^{BN}\right]\right)$ and an upper bound $b > \Psi(\delta, 0) = \frac{\delta}{1-\delta}\left(E\left[\pi^C\right] - E\left[\pi^{BN}\right]\right)$, the largest payoff differential can occur with positive probability for which the cartel always breaks up and for which the cartel never breaks up. As $\rho = 1$ and $\rho = 0$ can not be a fixed point, ρ must lie strictly between 0 and 1. ■

All elements combined, the benchmark rates cartel follows the feasible strategy of continuous collusion with episodic break-up:

$$\max_{\Delta v_{tt},\Delta c_{tt}} \sum_{i=1}^{N} \left((v_{0it} + \Delta v_{it}) \left(\frac{1}{n} \sum_{j \in \mathcal{T}} (c_{0jt} + \Delta c_{jt}) - L_{t-1} \right) - C(\Delta v_{it}, \Delta c_{it}) \right) |_{(v_{0t}, c_{0t})}$$

$$\text{if } \pi_{it}^{D} - \pi_{it}^{C} \leq \frac{\delta}{1-\delta} \left((1-\rho) E\left[\pi^{C}\right] + \rho E\left[\pi^{N}\right] - E\left[\pi^{BN}\right] \right) \quad \forall i = 1, ..., N, \tag{15}$$

$$\text{and } \max_{\Delta v_{it},\Delta c_{it}} \pi_{it} \quad \forall i = 1, ..., N \text{ otherwise.}$$

The cartel is characterized by occasional recourse to non-coordinated quoting to deal with extreme value exposure and eligible transaction rate drawings. This appears to be consistent with casual observation of the benchmark rate setting processes. In continuous collusion with episodic break-ups, the cartel strategy is not first-best, yet does minimize total costs of collusion. Note that while switches between collusion and break-up are discrete, the cartel agreement itself is continuous in that actual deviation is off-equilibrium. A stable cartel with episodic break-ups would also be sustainable under continuous adjustments, if it were viable.

4 Collusive Rate Patterns

To illustrate how benchmark rate collusion could play out in practice and the type of empirical trail such behavior would leave, we simulate a data generating process and determine the strategies of continuous collusion with episodic break-up. The cost function is specialized to

$$C(\Delta v_i, \Delta c_i) = \alpha (\Delta v_i)^2 + \beta (\Delta c_i)^2, \tag{16}$$

where α and β are the cost parameters for front running and eligible transactions rigging, respectively. The resulting linear-quadratic payoff function satisfies the conditions for a unique global maximum. Parameter values are: $N = 16$, $n = 8$, $\alpha = \beta = 1$ and $L_0 = 1$ is starting value. Portfolio positions and true borrowing costs are drawn from normal distributions, respectively $v_{0it} \sim N(0, 0.1)$ and $c_{0it} \sim N(L_{t-1}, 0.1)$.

First, the probability of break-up is calculated and the effect of the discount rate on cartel stability illustrated, resetting $L_{t-1} = 1$. We simulate draws of baseline exposures v_{0i} and baseline eligible transaction rate c_{0i}, derive payoffs in static Bayesian-Nash (π_{it}^{BN}), collusion (π_{it}^{C}), deviation (π_{it}^{D}) and static Nash (π_{it}^{N}) in each draw, for each bank $i = 1, ..., N$, and determine the expected payoffs $E[\pi^{BN}]$, $E[\pi^{C}]$ and $E[\pi^{N}]$.[79] Subsequently, the simulated distribution of the largest payoff differential $\max_i(\pi_{it}^{D} - \pi_{it}^{C})$ and the fixed point ρ as a function of discount rate δ are identified.

[79] Simulations are done in MATLAB®. The reported averages are based on 100.000 baseline draws v_{0it} and c_{0it} for each bank i. A detailed description the optimization methods is given in the appendix for the case of $N = 4$.

Second, these elements are used to generate time-series, which suggest possible empirical screening methods that could help tell collusion apart from independent quoting in practice.

4.1 Payoffs and Break-ups

The simulated payoff frequency distributions are Chinese hat-shaped around zero, yet a panel bank's average expected collusion payoff is almost thirty times higher than under independent quoting. Also the standard deviation of cartel profits is substantially higher, as there are many instances in which cartel members are obliged to take one for the team: making negative profits that are much larger than they would have achieved under independent quoting.[80]

Figure 5 provides a frequency table for the $\max_i \left(\pi_i^D - \pi_i^C\right)$ and identifies ρ as a function of δ. In the left-hand panel, $\delta = 0.90$ which provides the critical cut-off value $\Psi \approx 0.0027$. Together with a conditional expected collusion payoff, implying a break-up probability of around 0.39.[81]

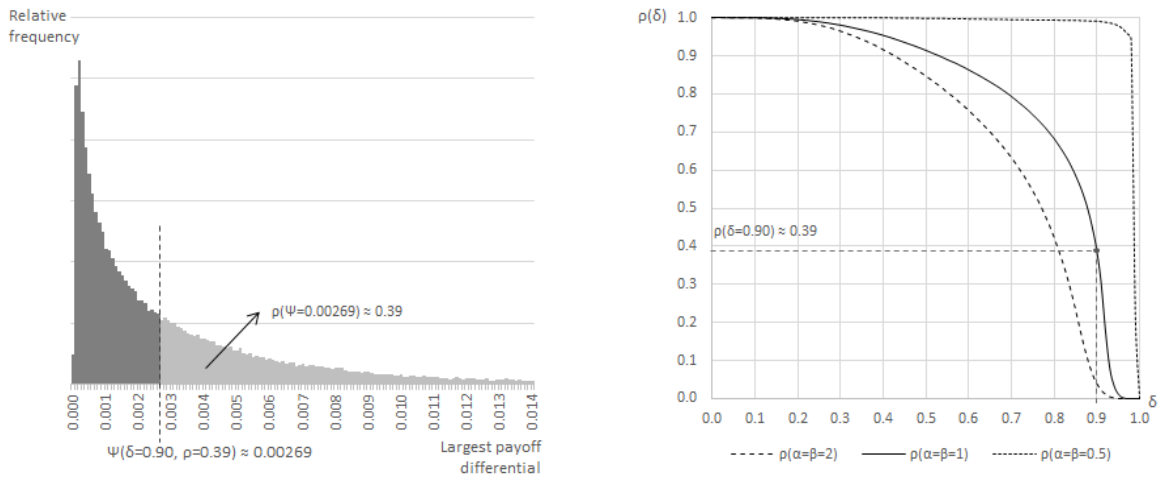

Figure 5: Left-panel: Frequency table of $\max\left(\pi_{it}^D - \pi_{it}^C\right)$ for $\delta = 0.90$. Right-panel: Cartel break-up probability ρ as a function of discount rate δ.

In the right-panel, the $\rho(\delta)$-curves are for three different levels of $\alpha = \beta$. The higher the cost of manipulation, the lower is the probability of break-up for all discount factors. Continuous collusion seems to be more stable when the cost of manipulation are high, which makes deviation less attractive, so that higher extreme value

[80] $E[\pi^{BN}] \approx 0.00003$, $\sigma_{BN} \approx 0.00071$; $E[\pi^C] \approx 0.00092$, $\sigma_C \approx 0.00686$; $E[\pi^N] \approx 0.00025$, $\sigma_N \approx 0.00296$.

[81] $E[\pi^C|\text{no break up}] \approx 0.00011$, $E[\pi^N|\text{break up}] \approx 0.00070$ and $\rho \approx 0.38778$.

positions can be sustained. With very low costs, the cartel breaks up constantly and is merely an exchange of information to play Nash rather than Bayesian-Nash. Note that since the reforms, the cost involved in manipulating Libor and Euribor have increased by requiring actual transactions in support of submissions, which would have resulted in less collusive break-ups. Further suggested reforms, such as extending the class of eligible transactions, may have a similar effect.

4.2 Time-series

Figure 6 displays the simulated interbank rate over time for $\delta = 0.90$, first when banks determine their submissions independently, respectively individually optimal and honest for 60 days each, and after that in continuous collusion with episodic break-up for 120 days. In the collusion period, the vertical shaded areas are episodes of non-cooperative quoting following an extreme value drawing—which happened 40 out of the 120 days of collusion, which is in the neighborhood of the 39% projected.

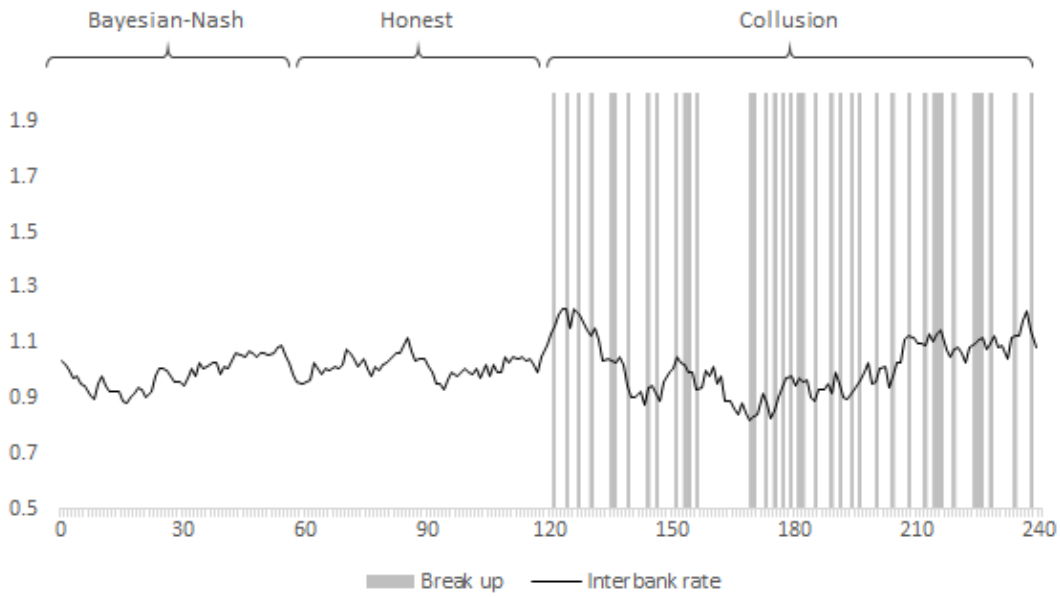

Figure 6: Simulated benchmark rate under Bayesian-Nash and honest independent quoting and collusion.

From the rates series alone it is not evident whether the banks quoted independently or collusively, nor which cartel periods were break-ups. The variance in the series appears to be similar, while any drift in the mean would be random hysteresis since the rate follows a random walk around 1. Two metrics could be used to identify the different periods: the intraday variance between the quotes of the panel banks

on any given day, and the interday variance (or volatility) of the interbank rate itself over a certain time window.

The intraday variance is displayed in Figure 7 for the full panel of 16 banks and the 8 banks in the trimmed range.[82]

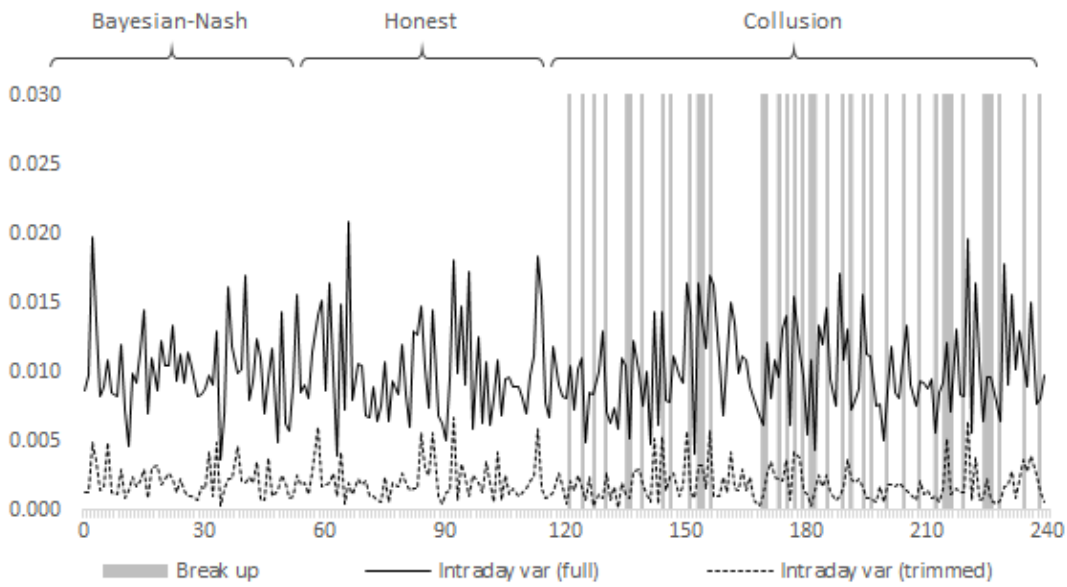

Figure 7: Intraday variance in quotes, full panel and trimmed range.

It does not obviously discriminate between the regimes. On analysis of Figure 1 in Section 2, Abrantes-Metz et al. (2012) conjecture that benchmark collusion would lead to reduced intraday variance. We do not find evidence for their conjecture in our model and simulations. While the trimming mechanism gives incentives to part of the trimmed banks to bunch around the pivotal quotes more extremely in the same direction than under independent quoting, which decreases the intraday variance, others quote their true borrowing costs, increasing the intraday variance. Even though, in line with evidence from the investigations, Abrantes-Metz et al. (2012) reject low-balling as the explanation of their observed period of low intraday variance, this seems to remain a possible explanation. It should also be noted that in their time window, there was considerable market turmoil.

Volatility in the quotes on the other hand does give distinct differences. Figure 8 shows the interday variance for two windows: 11 days and 5 days.

[82] On average, the intraday variance is 0.0099 for the full panel and 0.0020 for the trimmed range during the 60 honest days and 0.0101 for the full panel and 0.0019 for the trimmed range during the 120 manipulation days. These differences are not statistically significant. Also within the 120 manipulation days there is not significant difference between collusion days and temporary break-up days.

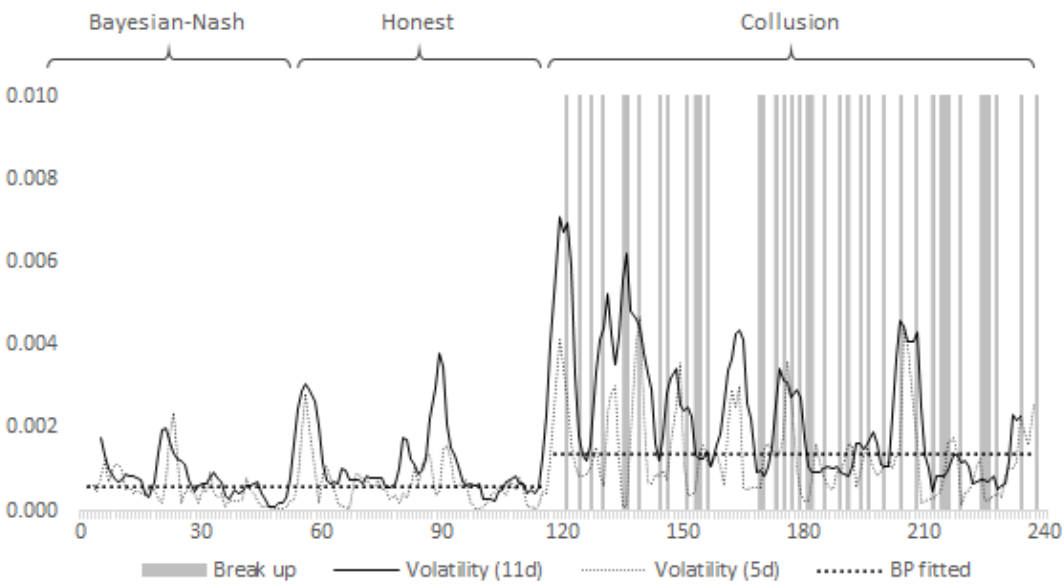

Figure 8: Volatility in quotes, 11 days and 5 days window, including 5-day Bai-Perron structural break test results.

Clearly, the benchmark rate under collusion displays more extreme behavior than during independent quoting—while again it is not possible to tell apart optimal Bayesian-Nash from honest independent quoting. The average volatility under collusive quoting is about twice as high as under independent behavior. This difference is statistically significant with a p-value below 0.00001, in both windows, using a one-sided Wilcoxon ranked sum test.[83] However, a volatility test cannot discriminate between no break-up and break-up during the collusive period.

Also the average absolute change in the interbank rate is statistically significantly different between the break-up and full collusion regimes at the 1% level.[84] Moreover, within the collusion period, the average absolute change in the interbank rate is statistically significantly higher in no break-up than during break-up.[85] Using a one-sided Wilcoxon rank sum test, the null that the mean of the volatility is the same during no break-up and break-up within the collusion period is rejected with a p-value of 0.0089. These results are robust against changes in the length of the rolling window.

[83] The average interday variance is 0.0012 for the 11-day window and 0.0007 for the 5-day window during the 60 honest days and 0.0023 for the 11-day window and 0.0013 for the 5-day window during the 120 manipulation days.

[84] On average, the absolute change in the interbank rate is 0.0228 during the 60 honest days and 0.0322 during the 120 manipulation days.

[85] The absolute change in the interbank rate is 0.0357 during collusion and 0.0253 during break-up.

The cartel benefits from more volatility in the benchmark rates over time, as that allows the panel bank members to exploit their inside information about the rates movements in advance by adjusting their portfolio exposures, against non-initiated financial institutions and investors. During break-ups, these benefits are much smaller, as cartel members no longer take into account the externality effects of their behavior. It can also cause them to pursue conflicting directional changes, reducing volatility. Note that if instead of the official (manipulated) Libor rate, the mean of the rate drawings were the true average borrowing costs, volatility is reduced, so that collusion may be harder to tell apart from independent quoting.

4.3 Screening

Our finding that the benchmark rate fluctuates more during periods of collusion (no break-up or break-up) than independent quoting (Bayesian-Nash or honest) can be exploited for the development of empirical screens that can help target deeper investigations. For application to actual data, with the periods of cartel and non-cooperative quoting unknown, Bai-Perron structural break tests can be used to identify collusion more systematically. Bai and Perron (1998, 2003) provide a collection of tests that allow for identifying structural changes, break dates and magnitudes of change in time-series when both the number and the dates of the breaks are unknown.[86] On the 5-day volatility, the Bai-Perron test identifies one and only one break occurring at day 117, which is close to the actual break day 120. The fitted values are drawn in Figure 8. Similar results are found using the 11-day volatility.

For application as rate manipulation detection tools in actual financial markets, such tests would obviously need to be further calibrated and controlled for other drivers of volatility in benchmark rates, in order to prevent type I and type II errors. If market events lead to a significantly different Libor in the next period, or a interday variance, a volatility screen could flag this as suspicious. Intraday variance screen have the advantage that they are robust to changes in the level of the Libor, but like the volatility screen, they are not robust to changes in volatility. To remedy this, structural changes in the volatility of the rates could be supplemented by tests on individual submissions and transaction patterns, for all of which a higher variance is expected under collusion than independent quoting. Periods of collusion would also leave structural traces in transactions over time, as all banks would change their exposure position in the same direction in which the rate is rigged, so that combined correlation tests could be applied.

While data on the interbank rates and individual submissions of panel banks (albeit embargoed for Libor) is readily available, transaction data availability appears to be limited at best. Eligible transactions, largely interbank loan data, could to a certain extent be retrieved from the TARGET2 real-time gross settlements system,

[86] For an application to identifying the begin and end dates of cartel effects, see Boswijk et al. (2017).

using a method such as the Furfine (1999) algorithm, for transactions within Europe. A similar data set, the Fedwire Funds Service, is the large-value bank payments system operated by the Federal Reserve banks in the United States. Kuo et al. (2013) develop a methodology to infer information about individual term dollar interbank loans settled through this system. However, the real challenge lies in identifying banks' overall exposure positions to the rate, as these are largely driven by OTC derivatives transactions, which take place without an exchange. Data for them is simply not currently available. Recent initiatives to systemically collect or provide access to bank transactions and construct Trade Repositories (TRs) aim at maintaining electronic records of all transactions data, including OTC derivatives transactions. These hold a promise of, if sufficiently developed in the future, providing authorities with the necessary transactions data on a sufficiently detailed level to be useful in screening for collusive benchmark rates fixing.

5 Damages

Possible collusion in the fixing of the benchmark rates will have various, large and widespread consequences, as the Libor, Euribor and forex rates are the fundament of financial and real markets around the world. As a result of the manipulation scandals, the benchmarks lost part of their trustworthiness as foundations of value and signals of underlying risks. With the exception maybe of some insiders, to many investors the uncoverings of the fraud must have come as a shock and the extent of it been unforeseen and incalculable. By adversely affecting overall trust in the financial system, there will have been consequences for financial market stability.[87] Arguably, as long as the panel banks would have succeeded to create the illusion of creditworthiness and so masked the severity of the financial crisis at the time, this may have kept the crisis at bay.[88] Eventually, however, such a conspiracy would have come out, certainly in these highly transparent financial markets, and then rather deepen the financial crisis instead. Moreover, machinations seem to have been going on before, and not quite with first intent to prevent systemic crises.

To the extent that movements in the pricing of futures, options and derivatives are zero-sum games, where there is a winning contract for every correlative losing one, the overall effect on welfare of the benchmark rates cartel may have been limited. This means that the manipulations would primarily lead to rent shifting, with little or no deadweight loss. Yet even with off-setting winners, each downside on a deal still potentially has an antitrust damage claim for being overcharged. The cartel gains could be at the expense of counterparty victims, non-panel banks and traders in

[87] See for example Duffie and Stein (2015) and statements by experts such as in Peterson Institute for International Economics, "Libor: A Question of Trust. Peterson Perspectives Interviews on Current Topics," 11 July 2012 and Bank of International Settlements, "Timothy Lane: Financial benchmarks—a question of trust," 24 March 2014.

[88] See Vaughan and Finch (2017), page 97.

financial products related to the benchmarks, including insurance companies, municipalities, corporations and investors who lent or borrowed at distorted rates tied to the Libor or other interbank rates. Using inside information to front-run on self-created variation, the cartel banks could shift their exposure position in their own favor, at the expense of other market participants.[89]

The loss of a reliable benchmark could have induced different borrowing behavior and thereby affect countless underlying commodity and services markets. By affecting market values, both of financial products and in underlying markets, the rates manipulations likely also impacted the efficient allocation of resources.[90] Too low a rate would have led lenders to withdraw funding, for example from mortgage markets or for small business, thus affected real family well-being and entrepreneurial activities that could have brought important benefits. Similarly, collusive prices in foreign exchange rates could have harmed international trade and foreign direct investments, if only because the forex cartel increased its members take, thus making transactions in foreign currencies more expensive, as well as postponing, or sometimes not executing, client orders that did not fit their own collusive profit objectives. These distortions could potentially have had rippling real effects on businesses and consumers worldwide.

Furthermore, the schemes in our model would introduce plain costs of manipulation, transaction costs but also foregone profits, for example, in the case of eligible transactions rigging, from trades at different rates than the going ones—or at different times than the commissioned ones—to support collusively intended future submissions—or currency value changes. In some scenario's, part of the cartel strategy could be to inflict high costs of collusion on designated cartel members, in order to keep internal stability. Yet, trades in eligibles would typically be small relatively to the total portfolio exposure. Some cartel agreements were found to be aimed at reducing transaction risks, maintaining narrower spreads for trades in order to lower the members' own transaction costs and maintaining liquidity between them.[91] The assessment of damages due to the benchmark rates cartels therefore will be highly case specific and often complex.

6 Concluding Remarks

Despite evidence of wider coordination in the benchmark rate fixings, the Libor and Euribor scandals have been passed off mostly as incidents of bilateral manipulation by a few rogue traders for their own benefit, quite possibly against the interests of their employer banks. In this paper we show how a full stable for-profit benchmark rates cartel could work, despite the design of the rates and the panel banks' interests

[89] See Foster (2014).

[90] See Abrantes-Metz et al (2012).

[91] European Commission, "Antitrust: Commission settles cartel on bid-ask spreads charged on Swiss Franc interest rate derivatives; fines four major banks € 32.3 million," 21 October 2014.

typically not being aligned. We reveal two mechanisms, observed in some of the government investigations, that can have facilitated this collusion: front running and eligible transactions rigging. By exchanging inside information, the panel banks could front run to create a more beneficial exposure position to the upcoming rate, thereby reducing conflicting interests in their trading books. Some cartel banks could also have engaged in eligible transactions rigging, placing (potentially suboptimal) transactions at rates required to allow the cartel to justify future quote submissions.

The collusion is costly, nevertheless worthwhile. Occasionally, participating banks would be required to incur manipulation costs exceeding the period cartel gains, leaving them losses. Even though these can be substantial, a panel bank's average expected collusion payoff is substantially higher than under independent quoting. Panel banks would often bear costs on the smaller book of eligible transaction for the occasional large gain on the full exposure when the rate is manipulated in particularly their favor. This picture seems broadly consistent with evidence on the money markets involved, in which panel members are in multi-market contact over a variety of financial products, typically meeting short-term inside liquidity demands also sometimes at a small loss, in order to maintain longer-term banking relationships and benefit from large outside business gains.

Consistent with the evidence found, the benchmark rates cartels are characterized by episodic recourse to independent quoting. We explain these temporary break-ups as part of an ongoing collusive strategy, to which the cartel reverts in response to occasional extreme exposure values that give incentive to deviate. These reflect that payoffs in financial markets can be volatile. We describe in detail how the cartels can be administered. The collusion leaves no obvious traces in the benchmark patterns over time, nor in intraday variance in the quotes. It does markedly increase the volatility in quotes between trading days. On this basis, we propose volatility screens, possibly supplemented with transactions data to collect, to monitor submissions for periods of collusive manipulation. Apart from forex, our model may also apply to other benchmarks—some of which have been subject to allegations of misconduct, such as in gold, energy and commodities markets.

Several of the assumptions we make warrant some further discussion. We model portfolio positions as independently distributed around zero, so that there is no accumulation and expectations on future positions are unrelated to current positions. Position fluctuations could also be around a steady bank-specific exposure profile, such as for example long-term mortgage contracts with a variable rate on the books, as long as their variance is large enough. Our model can then be seen as an approximation on the larger part of the portfolio, or alternatively as being about a traders cartel maximizing joint profits only on their own trading books in OTC derivatives, instead of their employer banks' overall exposures, in which case the panel banks as corporation would possibly be damaged on their remaining smaller portfolio parts.

Trade book building introduces highly complex additional optimization, as would expectations about future demand, correlation of demand shocks and other features

of business cycles, as in Haltiwanger and Harrington (1991), Kandori (1991) and Bagwell and Staiger (1997). In our model, such extensions have no obvious effect on cartel stability or cartel formation incentives. More complex shock dynamics is likely to introduce the necessity of side-payments or rotation on which banks are supposed to pay the eligible transactions rigging costs, for which there is some evidence.[92] It may also lead to longer break-up periods, as it may take time to get trading books incentives aligned again and find a stable cartel strategy. We leave analysis of collusive benchmark rates fixing with more complex portfolio dynamics for future research.

Instead of a full panel cartel, collusion could also take place in subsets of the panel banks only, for example as a result of bank heterogeneity, differences in supervisory regimes, or to avoid detection. Also, in case of a break-up, a full panel cartel may try to carry on as a partial cartel, with the bank(s) that drew an extreme value exposure not participating for the period. Allowing for subcoalitions likely makes the full cartel less stable, as banks that have the same interest as the partial cartel could be more inclined to leave for one period, knowing that the remaining cartel would submit a quote that is in line with their interests, while they could save the manipulation cost. The trimming mechanism on the other hand would ensure that at least a certain minimum cartel coalition would remain, as marginal contributions to the manipulated rate are high for the first couple of banks that get inside the trimmed range. Note also that in a full cartel the trimming mechanism reduces the cost of collusion as it allows part of the panel to quote their true borrowing cost.

Information sharing in the cartel is modeled to be perfect, which reveals to all members their options to deviate. Alternatively, the colluding banks could employ an independent administrator who collects the information, runs the cartel software and then only provides personal instructions to each bank—including the occasional break-up. While relying on such a cartel moderator would possible demand more trust from cartel members than can be expected amongst conspirators, if the panel banks would manage to all commit to such an administration, it would increase cartel stability by taking away incentives to deviate in large part.

The benchmarks remain vulnerable to the cartel mechanisms we suggest, also after the implementation of recent and proposed reforms. Moreover, further reforms to make the rates setting processes more resilient to collusion are not obvious. As shown, an extension of the class of transactions that are eligible to support rate submissions would increase the cost of manipulation, but at the same time lead to less break-ups and so potentially stabilize the cartel. Also the embargo on individual banks' Libor submissions does not affect a full cartel's ability to infer adherence to the collusive agreement from the rate itself—although it would be impossible to detect

[92] In Commodity Futures Trading Commission, "Order Instituting Proceedings: Deutsche Bank AG," 23 April 2015, on page 27 it is reported that: "The UBS Senior Yen Trader also offered to enter into trades at rates detrimental to him but beneficial to the Senior Yen Trader-Submitter to ensure the Senior Yen Trader-Submitter's involvement in his plans and to entice him to make Deutsche Bank' s Yen LIBOR submissions in the manner he desired."

deviation in partial cartels that consist of fewer members than the trimmed range plus the number of banks trimmed on one side. As done in Coulter and Shapiro (2015) and Duffie and Dworczak (2014) for individual manipulation incentives, a mechanism design approach could also help take away incentives to collude in benchmark rates fixing.

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A A MATLAB® Cartel Routine

The following MATLAB® script calculates the optimal cartel strategies. Each bank inputs its daily baseline values, with which the software derives the optimal collusion and deviation strategies and their associated payoffs. The routine also determines whether all of the N cartel stability conditions hold, and dictates break-up as a strategy to all cartel members when one or more do not. The script provides all banks with the exact front running and eligible transactions rigging strategies. The kernel is provided below—for the condensed case of $N = 4$.

```
% Parametric assumptions
N = 4; % Number of panel banks
n = 2; % Share of banks within trimmed range
a = 1; % FR cost parameter
b = 1; % ETR cost parameter
sv = 0.1; % Standard deviation exposure
sc = 0.1; % Standard deviation transaction rates
delta = 0.9; % Discount rate

% Derive critical cut-off level Psi
psi = fpsi[N,n,a,b,sv,sc,delta];

%% Step 1:  Prompt input baseline values
dlg_title = 'Enter baseline exposures'; num_lines = 1;
prompt = {'Bank 1','Bank 2','Bank 3','Bank 4'};
defaultans = {'','','','''};
V0 = str2double(inputdlg(prompt,dlg_title,num_lines,defaultans,'on'))';

dlg_title = 'Enter baseline transaction rates'; num_lines = 1;
prompt = {'Previous interbank rate',Bank 1','Bank 2','Bank 3','Bank 4'};
defaultans = {",",",",","};
C0 = str2double(inputdlg(prompt,dlg_title,num_lines,defaultans,'on'))';

%% Step 2:  Calculate collusion and deviation payoffs
% Collusion payoffs
fJointProfit = @(DC)-((sum(V0)+sum(DC(1,:)))*(trimmean(C0+DC(2,:),...
  n/N*100)-L0)-a*(sum(DC(1,:).^2))-b*(sum(DC(2,:).^2)));
CStrategy = fminunc(fJointProfit,zeros(2,N),options);
PCol = fpayoff(V0,C0,N,n,L0,a,b,CStrategy);

% Deviation payoffs
PDev = zeros(N,1);
for i = 1:N
```

```
 Cj = CStrategy; DCj(:,i)=[]; C0j = C0; C0j(:,i)=[];
 fOwnProfit = @(DD)-((V0(1,i)+DD(1,i))*(trimmean(horzcat(C0(1,i)+...
   DD(2,i),C0j+DCj(2,:)),n/N*100)-L0)-a*(DD(1,i).^2)-b*(DD(2,i).^2));
 DStrategy = fminunc(fOwnProfit,zeros(2,N),options);
 PDev(i,1) = fpayoffc(V0,C0,N,n,L0,a,b,CStrategy,DStrategy,i);
end

%% Step 3:  Check whether constraints hold and produce output
if PDev - PCol <= psi
 msgbox(sprintf(['Break-up:  No.',...
 '\n\n Advised positions:',...
 '\nBank 1:  Adjustment = %.4f, New position = %.4f',...
 '\nBank 2:  Adjustment = %.4f, New position = %.4f',...
 '\nBank 3:  Adjustment = %.4f, New position = %.4f',...
 '\nBank 4:  Adjustment = %.4f, New position = %.4f',...
 '\n\n Advised submission targets:',...
 '\nBank 1:  Adjustment = %.4f, New submission = %.4f',...
 '\nBank 2:  Adjustment = %.4f, New submission = %.4f',...
 '\nBank 3:  Adjustment = %.4f, New submission = %.4f',...
 '\nBank 4:  Adjustment = %.4f, New submission = %.4f'],...
 CStrategy(1,1), V0(1)+CStrategy(1,1), ...
 CStrategy(1,2), V0(2)+CStrategy(1,2), ...
 CStrategy(1,3), V0(3)+CStrategy(1,3), ...
 CStrategy(1,4), V0(4)+CStrategy(1,4), ...
 CStrategy(2,1), C0(1)+CStrategy(2,1), ...
 CStrategy(2,2), C0(2)+CStrategy(2,2), ...
 CStrategy(2,3), C0(3)+CStrategy(2,3), ...
 CStrategy(2,4), C0(4)+CStrategy(2,4)));

else
 msgbox('Break-up:  Yes.')
end
```

Given the parameters of the rate setting process, the cut-off value Ψ is found in routine `fpsi` that simulates a sufficient amount of daily payoff values in case of Bayesian-Nash, Collusion, Deviation and Nash (100.000 times in the results reported here), such that the fixed point ρ can be identified with sufficient precision. The Bayesian-Nash strategies are found by calculating the expected baseline values of the other $N-1$ banks and for each bank separately using the `fminunc` function in MATLAB® under the assumption that the calculated expected baseline values hold for the other banks. Nash strategies following break-up are found by each bank consecutively maximizing its own payoff function, repeated for a sufficient number of

rounds. Banks respond to each other for up to 24 rounds, after which either a non-cooperative equilibrium in pure strategies is reached, or none is concluded to exist, in which case the outcome of round 24 is taken as the mixed-strategy equilibrium drawing.

Step 1, at 0_t in the morning, all cartel members report their baseline drawings, exposures v_{0i} and eligible transactions rate c_{0i}, which are entered as inputs in the prompt as shown in Figure 9.

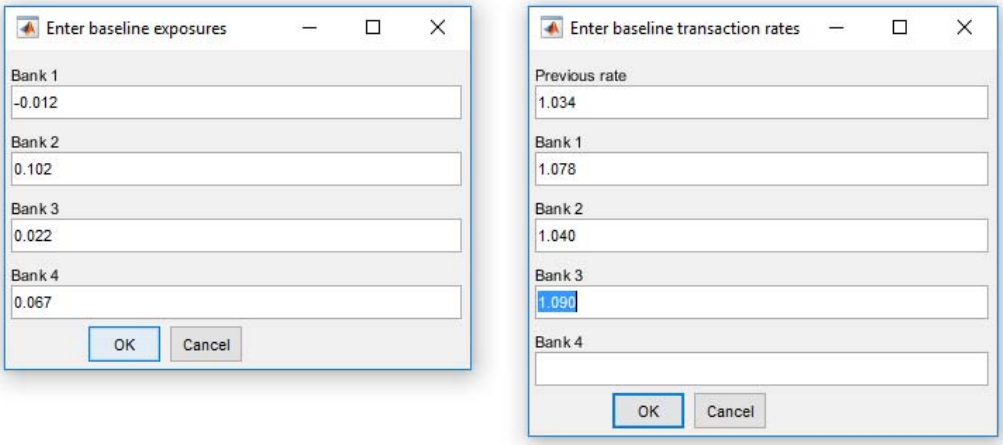

Figure 9: Baseline exposure and eligible transaction rate prompts (for $N = 4$).

In Step 2, the script subsequently derives the optimal cartel strategies, using the `fminunc` function. Taking `V0` as the $1 \times N$ vector of baseline exposures and `C0` as the $1 \times N$ vector of baseline eligible transaction rates, the code minimizes the objective function `ObjFunc` along the $2 \times N$ choice matrix `DC`, which represents the front running choice variables (first row) and eligible transaction rigging choice variables (second row). Note that `ObjFunc` is specified as the negative of the sum of the individual payoff functions, which is subsequently minimized. Output `CStrategies` are the optimal cartel strategies. Plugging these into the individual payoff functions provides each bank's cartel profits. This is done in the routine `fpayoff`. Similarly, the deviation payoffs are found by maximizing own payoffs given that other banks adhere to cartel strategy.

Finally, in Step 3 it is checked whether the difference between the deviation payoff and collusion payoff of each bank is below the critical cut-off value Ψ. The cartel instructions of all members are given to each, implying also all shared information, as illustrated in Figure 10. If none of the banks has a payoff differential above Ψ, the script reports that the cartel holds ('Break-up: No.') and which strategies each bank should implement. If at least one bank has a payoff differential above Ψ, all banks

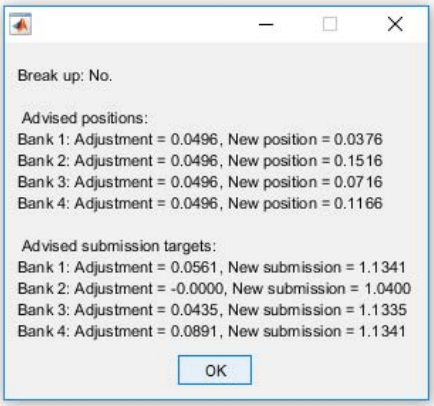

Figure 10: Cartel instructions (for $N = 4$).

receive the notification that collusive optimization is not stable ('Break-up: Yes.'), instructing them to revert to one-period static Nash.